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The research objectives were: (a) develop, test, and illustrate the use of a significant new approach to estimating the cost of an airframe production program using already collected data on Air Force airframes; and (b) provide the Air Force with a calibrated tool capable of providing timely answers to significant problems of program management.

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The researcher developed a model based on the four production cost drivers of learning by doing, learning over time, the speed of the production line, and production line length. It focuses on the production of an individual airframe as a function of its start date and its planned delivery date, and includes technical features of both the airframe production program and the contractor's behavior.

The model is estimated from data on the C-141 program, and is used to evaluate the effect of several small changes to the delivery schedule for the C-141. This analysis shows the sensitivity of the model to delivery schedule changes. It also illustrates one of the important ways that the model may be used in program management.

A detailed investigation of estimating the model on data from other programs revealed that its parameters are very stable from one program to another, and the parameters can be estimated from early actual data on a new program. Ways to combine the model with a cost estimating relation (CER) and update these estimates with early actual data are discussed in this report. These techniques are applied to data from the F-102 program and the F-5/T-38 program.

The report summarizes these results and discusses their applications to Air Force program management. This research can be used to understand production scheduling, aid program management, and estimate program costs.

Final Report

Cost Functions for Airframe Production Programs

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PREFACE

Clemson University has been engaged in active research on airframe cost estimation and production modeling since 1979. This research effort has resulted in numerous technical reports, published papers and professional presentations over the last three years. This report is the latest and most comprehensive of the series of reports.

This is the final report on our research funded by the Air Force Business Management Office under contract F33615-81-K-5116. This work was also funded in part by the Office of Naval Research under contract N00014-75-C-0451.

In the report we have summarized several technical reports produced in the past year. We have also attempted to point out areas in which these technical results have important applications in Air Force (and Navy) aircraft program management. Unfortunately the necessity to place this work in context, present the technical details and emphasize applications has led to a rather lengthy document. Those readers whose time and interest do not permit them to work through the mathematical details of the model derivations might choose to skip Chapters II, III and IV. The remainder of the document concentrates on background, rationale, results and applications.

SUMMARY

This research effort was to develop, test, and illustrate the use of a significant new approach to estimating the cost of an airframe production program. The theory was developed to unify previously separate methods of describing program costs. The effort was to result in a cost function that could be estimated from already collected data on Air Force airframes. We were to provide the Air Force with a calibrated tool capable of providing timely answers to significant problems of program management.

These objectives have been met by the revised model described in Chapter V. It is based on the four production cost drivers of learning by doing, learning over time, the speed of the production line, and production line length. This model focuses on the production of an individual airframe as a function of its start date and its planned delivery date. The model includes both technical features of the airframe production program and the contractor's behavior.

The revised model was developed after carefully considering numerous other models. These candidates were developed from theoretical principles. When the candidates proved tractable, they were evaluated with Air Force data. This model development and evaluation effort is described in Chapters II, III, and IV of the report.

The revised model is estimated from data on the C-141 program. The revised model fits this data very well. The

model is used to evaluate the effect of several small changes to the delivery schedule for the C-141. This analysis shows the sensitivity of the model to delivery schedule changes. It also illustrates one of the important ways that the revised model may be used in program management.

A detailed investigation of estimating the revised model on data from other programs revealed that the model's parameters are very stable from one program to another. The parameters can also be estimated well from early actual data on a new program. Also, ways to combine the model with a cost estimating relation (CER) and ways to update these estimates with early actual data are discussed in the report. These techniques are applied to data from the F-102 program and the F-5/T-38 program.

Chapter VIII of the report summarizes these results and discusses their applications to Air Force program management. There we point out how this research can be used to understand production scheduling, to aid program management, to estimate program costs, and as a basis for further research on production planning.

ACKNOWLEDGEMENTS

We wish to acknowledge the dedicated research and computational support of Jeff Camm to this effort. Andy Litteral also deserves acknowledgement for his computational support. Thanks are also due to Glenda Johns who typed (and frequently edited) the report.

CONTENTS

I. INTRODUCTION

A. Research Problem

Due to cost overruns, Congressional concern, and a continuing need for better planning estimates, it is imperative that new techniques be developed and old techniques refined to obtain better cost estimates for major weapon system production and acquisition. Along with these techniques, a better understanding of the factors and forces that determine cost is required. In particular, the sensitivity of program costs to alternative policy decisions must be accurately estimated if we are to meet the challenge of providing wise acquisition policy. Furthermore, the cost impacts of policy decisions must be readily available if they are to have an impact in the dynamic world of systems acquisition.

The problems of estimating the cost of military aircraft are legion. Current methods of estimating costs are: (a) the parametric method, which generates simple, imprecise estimates which are insensitive of many production decisions and (b) the "bottoms up" industrial engineering method, which generates complex, imprecise estimates which must be substantially revised if almost anything changes. Neither of these procedures offer much help to the program manager who must develop appropriate funding profiles, lot release dates and delivery schedules prior to program start. They offer even less aid to the program manager who must respond quickly to proposed changes in funding and schedules prior to and during a production program.

In contrast to the parametric method, this research involves modeling the factors that influence cost during an airframe production program. In particular, the influences of production rate, learning, and delivery schedules are studied. A modeling effort with this stated purpose requires considerable knowledge of both the planning and production stages in any airframe program.

At the outset of a production program, a tentative monthly production schedule for the program is negotiated between the contracting parties. This schedule permits planning for work force buildup, facility and tooling needs, and the ordering of long lead time items. This early situation is referred to as the planning stage.

Although the planned delivery schedule covers the life of the program, formal contractual agreements between the Department of Defense and manufacturers usually cover only one year's delivery requirements. Delivery requirements for subsequent years are funded through the exercise of options or separate contracts as funds are appropriated by the Congress. Over time, the situation tends to change. Funding in a particular year may be insufficient to cover the planned production, or a national emergency or changed mission requirements may argue for changes in production rate. This latter situation is referred to as the production stage.

Intuition, economic theory, and recently, empirical studies argue that production rate changes, at either stage in the program, affect program costs. In addition, Gaunt [9] points out that cost penalties for production rate changes are now embodied in some contracts.

The foregoing is generally accepted, but there is substantial disagreement about both the magnitude and the direction of the impact of production rate changes on program costs. Empirical studies of airframe programs in the last five years have documented cases where increases in production rate have been associated with increases, decreases, and no change in the unit cost of production for different airframes. (See for example Smith [29].) Our goal is to explain these contradictory results and unify these heretofore separate methods of describing program costs.

B. Background

The theoretical foundations of production rate impacts on cost are as old as the study of economics. Adam Smith's pin factory example [28] is an early statement of the effect. More recently, Asher [3] recognized the potential importance of production rate to aircraft production costs. But he could find little statistical support for the idea. Since 1956 the idea of combining learning effects and production rate effects in the explanation of aircraft costs has proceeded along two rather separate routes.

In 1959 Alchian [1] provided some theoretical observations concerning the interaction of learning and production rate. His paper was followed by Hirshleifer's 1962 discussion [13]. Preston and Keachie [25], Oi [21] and Rosen [26] also made contributions. All these papers added to our understanding of the process by which learning interacted with production rate to impact cost; but they were conceptual and almost completely data free.

Furthermore, for the most part, they generated results that were far too general for statistical estimation.

The second line of development has been mainly empirical. Ever since Alchian's 1959 paper [1], Rand Reports [7, 15, 16, 17] on aircraft cost estimation have attempted to include both volume and production as independent variables in their cost estimating relations. In his 1963 paper, Alchian [2] reports this attempt as early as 1948. Even though Alchian argues that both variables should be important, the resulting empirical work credits production rate with little, if any, explanatory ability. In fact, a recent study [17] concludes:

> In general, however, we must conclude that for predicting the overall effect of production rate on aircraft cost, generalized estimating equations based on statistical analyses of our sample of military aircraft would be too unreliable to be useful.

The Rand studies have been cross-sectional studies characterized by a few observations on many aircraft programs. More recent work by Womer [37], Smith [29] and his students, Congleton and Kinton [4] has reached the opposite conclusion. The studies under Smith's direction have been time series studies on single airframe programs. Unfortunately, these studies have been almost devoid of economic theory. As a result even though some of the studies indicate that production rate is correlated with costs on a program, our understanding of the process by which this happens is fuzzy at best. Without this knowledge the results cannot be intelligently used for policy guidance.

C. Recent Work

Recently work has been aimed at closing the gap between these two lines of research. Washburn [33] and Womer [38] derive cost relations consistent with economic theory in forms suitable for empirical estimation. This work shows that, in the absence of outside forces, the producer attempting to minimize cost will change the production rate over time. That is, not all changes in production rate on weapon systems' programs can be regarded as resulting from government action. These results refute some previous theoretical work as pointed out by Womer [39]. They also provide potential explanation for Smith's [29] seemingly contradictory results.

At the same time, the unique data problems of combining variables measured by time periods with others measured by units produced have been examined by Womer [37]. Preliminary work has also related Womer's [37] model to the Rand data. Womer's model [34] has also been modified to include previous production experience and monthly production targets. This permits a production program to be modeled as a series of discrete tasks connected by experience. The impact of an exogenous increase or decrease in deliveries is therefore modeled. Likewise, the impact of stretching a lot over a longer period of time is modeled by this procedure.

Next the model was expanded to include the impact of several restrictions on production. For example, the impacts of constant work force restrictions and of other resource constraints like tooling capacity were examined. This permits a more realistic distinction to be drawn between the less restricted planning situation and the more restricted production situation. This work resulted in Womer [35].

Orsini's data on the C-141 program [22] was compared to the OSD data [20] on the same program. Discrepancies between the two data sets were resolved and the data was transformed into observations on the model. Then several versions of the model were estimated with the C-141 data. Preliminary results of this estimation were reported in Womer and Gulledge [34]. This resulted in a very good fit to the C-141 data, but the study also raised some interesting questions which deserved further study. In particular, questions about the best form of the underlying airframe production function and about the appropriate statistical model required answers. These recent efforts were the foundation for the present study.

D. Approach

Our goal of unifying the previously separate methods of describing program costs led us to adopt an approach more general than either Alchian's or Smith's. In particular, we make use of actual delivery schedules, not lot averages as Smith did, to explain program costs. This permits us to investigate the sensitivity of program cost to changes in the schedule which do not change the average delivery rate or the average manufacturing rate.

A second distinguishing characteristic of our research is our explicit emphasis on contractor behavior. All the models reported in the sections which follow have been derived from the assumption that contractors are motivated to choose ways of producing that minimize program costs. In particular, we start with the hypothesis that it is not only possible, but likely, that the contactor will choose a plan of production that calls for production rates and labor use rates to vary over the life of the program. This is more general than the approach of Alchian [1], who required constant production rates. This explicit emphasis on contractor behavior also permits the adaptation of our models to investigations where different behavioral assumptions might be made. In particular, if alternative contract types induce different contractor behavior, this approach can be used to investigate the cost impact of alternative contract types. The models which result from this approach are consistent with earlier findings which, in turn, seem to contradict each other. They are also consistent with accepted learning curve theory. They are more explicit in their treatment of both delivery schedules and contractor behavior, and they can be estimated with good precision early in an airframe production program. In the process we have also discovered several other important questions which need to be addressed by production research as more detailed data become available. An overview of the work accomplished on this contract is described next.

E. WORK ACCOMPLISHED

The work on this contract was divided into four tasks as listed in Table I.

TABLE I

Tasks

1. Conduct residual analysis on the C141 model and trace the implication of the results for the production function.
2. Determine an appropriate method to estimate the cost function from early actual observations on the program.
3. Validate the model and procedures on other program data.
4. Write the final report.

The first task was to conduct a detailed residual analysis and sensitivity analysis on the C-141 airframe model described in [34] and to trace the implications of the analysis results on the underlying production function.

The residual analysis and the sensitivity analyses are reported in Chapter II. They result in a clearer understanding of the forces at work in a production program that we have summarized as four effects of scheduling on production.

These analyses imply that particular generalizations of the production function might be useful. They led us to the dynamic multiple product production functions modeled in Chapter III. In Chapter IV estimates of the resulting model from the F-102 data are reported.

The multiple output production functions improved our insight into the production process; but only one proved tractable for our purposes. Unfortunately, the data available do not seem to be rich enough to provide reliable estimates of its parameters. Therefore, a different route was followed to yield a revised production model. This model is reported in Chapter V. Chapter V also includes estimates of the revised model on the C-141 data. It concludes with a sensitivity analysis which illustrates the use of the revised model in program decisionmaking.

In Chapter VI procedures for applying the revised model to early actual observations on a program (Task 2) are investigated. First the revised model, estimated from F-102 data is reported. Then attempts to estimate the model from early C-141 data are reported and evaluated. Combining the model with a cost estimating relation (CER) is also investigated. Furthermore, procedures to use a model estimated on one program's data together with early observations on a new program are investigated. The significance of these results is also discussed in Chapter VI.

In Chapter VII the results of our validation effort (Task 3) are reported. Here the revised model is applied to the F-5/T-38 program. The ability of the model to describe that program is evaluated. In Chapter VII the procedures reported in Chapter VI are also applied to the F-5/T-38 program. These results form the basis for our conclusions at the end of the chapter.

Chapter VIII describes ways in which the models and procedures developed in this study may be used in program management and cost estimation. There the study is briefly summarized, applications of the results to Air Force problems are discussed, and conclusions are drawn.

II. ANALYSIS OF THE C-141 MODEL

A. Introduction

In this chapter the basic model for the C-141 is analyzed. Here the C-141 program and data are described, assumptions are stated and the C-141 model is developed and estimated. This model was developed prior to the start of the current research effort. It is reported in Womer and Gulledge [34]. Task one of this contract calls for a residual analysis of the model and a detailed sensitivity analysis. These analyses are reported in section E of this chapter. The chapter is concluded with a discussion of the four effects which these analyses suggest are important in production planning.

B. The C-141 Program and Data

The C-141 program produced 284 aircraft during the six year period from 1962 to 1968. Only one model of the aircraft was produced. Data for this study is drawn from two sources. Orsini [22] reports direct man-hours per quarter for each of the twelve lots in the C-141 program. He also reports a delivery schedule for the aircraft by month. Orsini attributes these data to the C-141 Financial Management Reports maintained by the Air Force Plant Representative Office located at the Lockheed-Georgia facility. The schedule of actual aircraft acceptances by month as reported in the OASD (PA&E) publication Acceptance Rates and Tooling Capacity for Selected Military Aircraft [20] was used to check the Orsini delivery data.

This data, like much data on aircraft production, provides labor hours for a period of time (quarterly) and dates and quantities of deliveries. Unfortunately, there is no available information which relates output to the period of time over which labor hours are observed. One approach to this problem, used by Orsini, is to make some assumption about the pace of production on the program and aggregate the qu rterly data across lots. In addition to being arbitrary, this approach reduces 91 potential observations to 24. Our approach to the data problem is to construct a detailed production model of the aircraft to be delivered in any month. We then aggregate the model to expain the data rather than the other way around.

Preliminary analysis of the data revealed two additional data problems. First, there were two instances, late in the program, where a small number of labor hours were expended on a production lot after the schedule indicated delivery. This probably is a situation where deliveries were made out of sequence. To remedy this problem the labor hours for the last quarter of lots 9 and 10 were aggregated with those of the previous quarter. This reduced the number of observations by two.

The other problem is that in lots two through eight, delivery of the aircraft seems to lag the last expenditure of labor hours by an average of four months. For the other five lots labor hours are expended up to the last month of delivery. To overcome this problem, the deliveries of aircraft in lots two through eight were advanced by four months.

With these adjustments eighty-nine observations on labor hours for twenty-four quarters for twelve lots were used. These observations, together with the number of aircraft delivered each month, constitute the data for the study.

C. The Model

The model augments a homogeneous production function with a learning hypothesis. The discounted cost of production is minimized subject to a production function constraint, and the optimal time path of resource use is derived. Since factor prices are assumed to be constant over the relevant time period, cost is measured in the units of the variable resource. The variables used in the analysis are:

i = an index for a batch of airframes in the same lot (j) all of which are to be delivered at time t_{ij},

n_j = the number of batches in lot j,

m = the total number of lots in the production program,

D_{ij} = the number of airframes in batch i of lot j,

E_{ij} = a measure of experience prior to beginning batch i, the cumulative number of airframes to be delivered,

$$\text{i.e., } E_{ij} = \sum_{k=1}^{j-1} \sum_{h=1}^{n_j} D_{hk} + \sum_{h=1}^{i-1} D_{hj},$$

t_j = date work begins for all the batches of lot j,

t_{ij} = date work ends for batch i of lot j,

$q_{ij}(t)$ = production rate at time t on batch i of lot j,

$Q_{ij}(t)$ = cumulative production on batch i of lot j at time t,

$$\text{i.e., } \int_{t_j}^{t} q_{ij}(\tau)\, d\tau,$$

$x_{ij}(t)$ = rate of resource use at time t on batch i of lot j,

δ = a parameter describing learning prior to batch i,

ε = a parameter describing learning on batch i,

γ = a parameter describing returns to the variable resources,

α = a parameter associated with decreases in labor productivity as a batch of airplanes nears completion,

ρ = the discount rate,

C = discounted variable program cost,

C' = discounted variable costs for a single batch of airframes.

The production function is assumed to be of the following form:

$$q_{ij}(t) = A\ E_{ij}^{\delta} Q_{ij}^{\varepsilon}(t)\ x_{ij}^{1/\gamma}(t)\,(t_{ij}-t)^{\alpha} \tag{2.1}$$

where A is a constant. The input x is assumed to be a composite of many inputs whose use rate is variable throughout the production period. The output rate for any batch is related to the rate of resource use, cumulative output previous to batch i, and cumulative experience during the production of a given batch. The additional factor, $t_{ij}-t$, is included to compensate for changes in productivity that occur as the batch delivery date is approached. This reflects the gradually changing tasks from part manufacturing, fabrication, and assembly to testing during the production process. If $\alpha > 0$ as we expect, the effect of this term is to make it more difficult to increase production rate on a batch of airframes close to their delivery by adding resources. This is consistent with our assumption that, relative to early manufacturing tasks, system integration and testing tasks are time consuming rather than resource consuming.

The production function (2.1) is assumed to be homogeneous of degree $1/\gamma$ in the resources with $\gamma > 1$. Also, it is assumed that the technological change induced by experience is Hicks neutral. This avoids having to state a different learning hypothesis for each of the variable factors included in the composite resource. In particular, we assume that the material learning rate and the labor learning rate are equal.

Although the objective of the firm is a function of the wording of the contract, one goal of most contracts is to induce the firm to minimize discounted cost. The problem may be stated as:

$$\text{Min } C = \sum_{j=1}^{m} \sum_{i=1}^{n_j} \int_{t_j}^{t_{ij}} x_{ij}(t) e^{-\rho t} dt \tag{2.2}$$

$$\text{ST:} \quad q_{ij}(t) = AE_{ij}^{\delta} \, Q_{ij}^{\varepsilon}(t) x_{ij}^{1/\gamma}(t)(t_{ij}-t)^{\alpha}, \qquad \begin{array}{l} i = 1, 2, \ldots, n_j \\ j = 1, 2, \ldots, m \end{array}$$

$$Q_{ij}(t_{ij}) = D_{ij}, \qquad i = 1, 2, \ldots, n_j$$

$$Q_{ij}(t_j) = 0.$$

Since total cost is monotone nondecreasing and the sub-problems are additive, the solution to (2.2) can be obtained by minimizing each of the sub-problems. The problem may then be stated as:

$$\text{Min } C' = \int_{t_j}^{t_{ij}} x_{ij}(t) e^{-\rho t} dt \tag{2.3}$$

$$\text{ST:} \quad q_{ij}(t) = AE_{ij}^{\delta} \, Q_{ij}^{\varepsilon}(t) x_{ij}^{1/\gamma}(t)(t_{ij}-t)^{\alpha},$$

$$Q_{ij}(t_{ij}) = D_{ij}$$

$$Q_{ij}(t_j) = 0.$$

This problem is an optimal control problem which may be solved directly by minimizing the Hamiltonian function. However, the problem can easily be transformed into the problem of Lagrange, which can be solved using classical variational techniques. At this point the redundant ij subscripts are dropped. Solving the constraint for x(t) yields

$$x(t) = q^{\gamma}(t)A^{-\gamma}E^{-\gamma\delta}Q^{-\gamma\varepsilon}(t)(t_{ij}-t)^{-\alpha\gamma}. \tag{2.4}$$

A transformation is desired that yields one state variable and one control variable, the control variable being the time rate of change of the state variable. Let

$$Z(t) = A^{-1}E^{-\delta}Q^{1-\varepsilon}(t)/(1-\varepsilon). \tag{2.5}$$

This implies that

$$z(t) = A^{-1}E^{\delta}Q^{-\varepsilon}(t)q(t). \tag{2.6}$$

For the transformed problem, Z(t) will be the new state variable, and its time derivative, z(t), will be the control variable. Formation of the new objective functional requires absorbing the constraint. After using (2.4) and (2.6), an expression is obtained for x(t) in terms of the new control variable, i.e.,

$$x(t) = z^{\gamma}(t) \quad (t_{ij}-t)^{-\gamma\alpha}. \tag{2.7}$$

Substituting into the objective functional and setting the boundary condition yields the following transformed problem:

$$\text{Min } C' = \int_{t_j}^{t_{ij}} z^{\gamma}(t)\ (t_{ij}-t)^{-\gamma\alpha} e^{-\rho t} dt = \int_{t_j}^{t_{ij}} I\ (z,t) dt \quad (2.8)$$

ST: $Z(0) = 0,$

$$Z(t_{ij}) = A^{-1}E^{-\delta}D^{1-\varepsilon}/(1-\varepsilon).$$

Since the intermediate function, I, does not depend explicitly on the state variable, the Euler equation is

$$\frac{\partial I}{\partial z} = \gamma z^{\gamma-1}(t)\ (t_{ij}-t)^{-\gamma\alpha} e^{-\rho t} = K_0. \quad (2.9)$$

Solving for optimal z(t) yields

$$z(t) = K_1\ (t_{ij}-t)^{\gamma\alpha/(\gamma-1)} e^{\rho t/(\gamma-1)}. \quad (2.10)$$

This also provides a solution for the optimum time path of resource usage, i.e.,

$$x(t) = K_1^{\gamma}(t_{ij}-t)^{\alpha\gamma/(\gamma-1)} e^{\gamma\rho t/(\gamma-1)}. \quad (2.11)$$

This optimal solution to the problem is only of transient significance since the value of the constant K_1 is unknown. What is needed is an optimal expression for x(t) that is in terms of the variables and parameters of the original problem. To obtain the constant, notice that

$$Z(t) = \int K_1 (t_{ij}-t)^{\alpha\gamma/(\gamma-1)} e^{\rho t/(\gamma-1)} dt + K_2. \qquad (2.12)$$

Let $v = \rho(t_{ij}-t)/(\gamma-1)$, then

$$Z(v) = \int K_1 \frac{(\gamma-1)}{\rho} v^{\alpha\gamma/(\gamma-1)} e^{-v+\rho t_{ij}/(\gamma-1)} J dv + K_2 \qquad (2.13)$$

where J is the Jacobian of the transformation. Now, $u(t_j) = \rho(t_{ij}-t_j)/(\gamma-1)$ and $u(t_{ij}) = 0$, so choosing 0 and u as the limits of integration the appropriate integral is

$$Z(u) = K_3 \int_0^u v^{\alpha\gamma/(\gamma-1)} e^{-v} dv + K_4. \qquad (2.14)$$

Integration of this expression yields

$$Z(u) = K_3\, \Gamma[u, \alpha\gamma/(\gamma-1)+1] + K_4$$

where Γ is the incomplete gamma function. To satisfy the initial condition that $Z[u(t_j)] = 0$, let

$$Z(u) = -K_3\, \{\Gamma[\rho(t_{ij}-t_j)/(\gamma-1), \alpha\gamma/(\gamma-1)+1] - \Gamma[u, \alpha\gamma/(\gamma-1)+1]\} \qquad (2.15)$$

Also let

$$-K_3 = A^{-1} E^{-\delta} D^{1-\varepsilon} (1-\varepsilon)^{-1} \Gamma^{-1}[\rho(t_{ij}-t_j)/(\gamma-1), \alpha\gamma/(\gamma-1)+1], \qquad (2.16)$$

then Z also satisfies the final condition $Z(t_{ij}) = A^{-1}E^{-\delta}D^{1-\varepsilon}/(1-\varepsilon)$. Also note that

$$z(t) = \frac{dZ(u)}{dt} = K_3\ [\rho\ \frac{(t_{ij}-t)}{(\gamma-1)}\ ]^{\alpha\gamma/(\gamma-1)} e^{-\rho(t_{ij}-t)/(\gamma-1)} (\frac{-\rho}{\gamma-1}). \quad (2.17)$$

After substituting for K_3, the following expression is obtained:

$$z(t) = A^{-1}E^{-\delta}D^{1-\varepsilon}(1-\varepsilon)^{-1}\ \Gamma^{-1}\ [\frac{\rho(t_{ij}-t_j)}{(\gamma-1)},\ \frac{\alpha\gamma}{\gamma-1} + 1]$$

$$[\frac{\rho(t_{ij}-t)}{(\gamma-1)}]^{\alpha\gamma/(\gamma-1)}\ e^{-\rho(t_{ij}-t)/(\gamma-1)}\ (\frac{\rho}{\gamma-1}). \quad (2.18)$$

This formulation for optimum z(t) along with (2.10) provides a direct solution for K_1. Substitution for K_1 in (2.11) yields the following optimum time path of resource use:

$$x(t) = A^{-\gamma}E^{-\gamma\alpha}D^{\gamma(1-\varepsilon)}(1-\varepsilon)^{-\gamma}\Gamma^{-\gamma}[\rho(t_{ij}-t_j)/(\gamma-1),\ \alpha\gamma/(\gamma-1) + 1]$$

$$[\rho/(\gamma-1)]^{\alpha\gamma^2/(\gamma-1)+\gamma} e^{-\gamma\rho t_{ij}/(\gamma-1)} (t_{ij}-t)^{\alpha\gamma/(\gamma-1)} e^{\gamma\rho t/(\gamma-1)} \quad (2.19$$

This is the optimum time path of resource use for any given batch of airframes. Notice that this specification is not appropriate for the first batch within the first lot, i.e., $E_{ij} = 0$. But, since the first batch in the C-141 data contained only one airframe, the impact of the omission on total program cost is slight.

Since the data presented in the C-141 study is quarterly data, the quantity of interest is the total resource use over a quarterly period. If T and T-1 represent the beginning and ending dates for the quarterly period for some batch i, the appropriate expression is:

$$X(T)-X(T-1) = \int_{T-1}^{T} x(t)dt, \tag{2.20}$$

and using (2.11) the integral is

$$X(T)-X(T-1) = \int_{T-1}^{T} K_1{}^{\gamma}(t_{ij}-t)^{\alpha\gamma/(\gamma-1)}e^{\gamma\rho t/(\gamma-1)}dt. \tag{2.21}$$

Let $y = \gamma\rho(t_{ij}-t)/(\gamma-1)$, then

$$X(T)-X(T-1) = -K_1{}^{\gamma} \int_{\gamma\rho(t_{ij}-T-1)/(\gamma-1)}^{\gamma\rho(t_{ij}-T)/(\gamma-1)} [(\gamma-1)/\gamma\rho]^{\alpha\gamma/(\gamma-1)} y^{\alpha\gamma/(\gamma-1)}e^{-y}e^{\gamma\rho t_{ij}/(\gamma-1)}[-(\gamma-1)/\gamma\rho]dy. \tag{2.22}$$

Notice that this is a form of the incomplete gamma function. Integration of (2.2) yields

$$X(T)-X(T-1) = K_1{}^{\gamma}[(\frac{\gamma-1}{\gamma\rho})^{\alpha\gamma/(\gamma-1)+1}]\, e^{\gamma\rho t_i/(\gamma-1)}$$
$$\{\Gamma[\gamma\rho(t_{ij}-T+1)/(\gamma-1),\alpha\gamma/(\gamma-1)+1] - \Gamma[\gamma\rho(t_{ij}-T)/(\gamma-1),\alpha\gamma/(\gamma-1)+1]\}. \tag{2.23}$$

Substituting for K_1 and performing the necessary algebra leaves an expression that represents the optimum amount of resource use over an interval of time for an individual batch of airframes, i.e.,

$$X_{ij}(T)-X_{ij}(T-1) = A^{-\gamma}E_{ij}{}^{-\delta\gamma}D_{ij}{}^{\gamma(1-\varepsilon)}(1-\varepsilon)^{-\gamma}$$
$$\Gamma^{-\gamma}[\rho(t_{ij}-t_j)/(\gamma-1),\alpha\gamma/(\gamma-1)+1]\ \left(\frac{\rho}{\gamma-1}\right)^{\gamma(\alpha+1)-1}$$
$$\gamma^{-\alpha\gamma/(\gamma-1)-1}\{\Gamma[\gamma\rho(t_{ij}-T+1)/(\gamma-1),\alpha\gamma/(\gamma-1)+1] -$$
$$\Gamma[\gamma\rho(t_{ij}-T)/(\gamma-1),\alpha\gamma/(\gamma-1)+1]\}. \tag{2.24}$$

However, because of the nature of the data it is impossible to observe the quantity on the left side of equation (2.24). What is observable is direct man-hours per lot. This means that the observed quantity is

$$\sum_{i=1}^{n_j} [X_{ij}(T)-X_{ij}(T-1)]$$

where there are n_j batches in a lot. For this study, the sum is the observed values of labor hours that are reported in Orsini's data set. This sum and the number of aircraft delivered each month are the variables that are used to empirically test the validity of the model.

D. Empirical Results

To explore the applicability of the theoretical specification, the parameters in (2.24) are estimated using Orsini's C-141 data. Let

$$\beta_0 = A^{-\gamma}(1-\varepsilon)^{-\gamma}[(\rho/(\gamma-1)]^{\alpha\gamma+\gamma-1}\gamma^{-\alpha\gamma/(\gamma-1)-1}$$

and

$$\beta_1 = \alpha\gamma/(\gamma-1) + 1.$$

The model may be restated as:

$$\sum_{i=1}^{n_j} X_{ij}(T)-X_{ij}(T-1) = \sum_{i=1}^{n_j} \beta_0\, E_{ij}^{-\gamma\delta} D_{ij}^{\gamma(1-\varepsilon)}$$
$$\Gamma^{-\gamma}[\rho(t_{ij}-t_j)/(\gamma-1),\ \beta_1]$$
$$\{\Gamma[\gamma\rho(t_{ij}-T+1)/(\gamma-1),\ \beta_1]$$
$$-\Gamma[\gamma\rho(t_{ij}-T)/(\gamma-1),\ \beta_1]\}. \tag{2.25}$$

Since the monthly delivery dates for each batch within each lot are known, it is possible to estimate the parameters in (2.25) using nonlinear least squares. Proc NLIN of the Statistical Analysis System [27] was used for this purpose.

Initially, the value of the discount rate was assumed to be .025. (Since time is measured in quarters this corresponds to a 10% yearly discount rate.) The remaining parameters were estimated using Marquardt's compromise. Diagnostic checking revealed that the estimates for ε and δ were extremely collinear. This suggests that an alternative specification with ε restricted to be equal to δ would be appropriate. Also, the restriction that

ρ = .025 was relaxed, and ρ was estimated simultaneously with the other parameters in the model.

The results of both regressions are presented in Table II. All of the parameter estimates are significantly different from zero, and the signs agree with a priori expectations.

Table II

Parameter Estimates and Asymptotic Standard Errors

	Parameter Estimates ρ .025		Parameter Estimates ρ estimated from the Data	
	Estimate	Standard Error	Estimate	Standard Error
β_0	5.755	.9875	5.839	1.0173
β_1	3.287	.1288	3.163	.5267
$\delta=\varepsilon$	.2733	.0274	.272	.0271
γ	1.019	.0004	1.041	.0004
ρ	.025	*	.049	.0096
	MSE = 3.92×10^{10}		MSE = 3.97×10^{10}	

*The Standard Error is not estimated since ρ is fixed.

In particular, notice that the value of γ is significantly greater than one, indicating that the production function does exhibit decreasing returns to the variable factor. The estimated value of the learning parameter is also consistent with a priori expectations. A $\delta=\varepsilon$ value of .272 is consistent with an 83% learning curve. In addition, the estimate for $\beta_1 = 3.163$ implies that $\alpha = .085$ which is positive as expected.

E. Analyses

The analysis of residuals for the model was accomplished by plotting the estimated error terms from the model versus quarter of observation by lot. This analysis indicated that the statistical assumption of independent error terms was met reasonably well.

Tests for autocorrelation were inconclusive, but there did seem to be some heteroscedasticity in the error terms. However, neither of these problems seemed to be quantitatively important enough to cause major biases in the estimates.

The sensitivity analysis of the model was more interesting. The estimated relation for labor use on lot j in the quarter that ends at time T is

$$R_{jT} = \sum_{i=1}^{n_j} 5.84\, E_{ij}^{-0.283} D_{ij}^{0.758} \Gamma^{-1.041}[1.2(t_{ij}-t_j),\ 3.16]$$

$$\{\Gamma[1.24(t_{ij}-T+1),\ 3.16] - \Gamma[1.24(t_{ij}-T),\ 3.16]\} \qquad (2.26)$$

where

$$R_{jt} = \sum_{i=1}^{n_j} X_{ij}(T) - X_{ij}(T-1) \qquad (2.27)$$

Equation (2.26) describes labor required as the sum of the labor required for each of the batches of aircraft in lot j. This is the form of the model estimated by NLIN.

Given the estimated parameters, the estimated time path of resource use rate for a batch of airframes (2.19) may be found. Adding this over all the batches in a lot yields the time path of resource use rate for the lot. These estimated relations are illustrated for a sample lot in Figure 1. In Figure 1 the areas under each curve to the left of any point in time show the labor required up to that time to support the indicated delivery. The time path of labor required for the lot is the sum of these requirements. This is indicated by the curve with the largest area. The figures show that resource use rises at an increasing rate from time t_j to an inflection point, after which it continues to rise, but at a decreasing rate. Eventually resource usage reaches a maximum and declines thereafter. We attribute the eventual decline in resource use rate to a decrease in the marginal product of labor as the delivery date for a batch approaches. That is, before components of the airframe are assembled, adding more labor easily increases the production rate; but after most of the components are assembled, crowding makes it more difficult to increase production rate on a batch by adding more labor. In fact, the rate of labor use on the batch must fall to provide the optimal production rate. In addition, the time consuming testing procedures that preceed delivery may not be able to be compressed by more labor. Therefore, there is a period of time near the end of a project where labor use is significantly

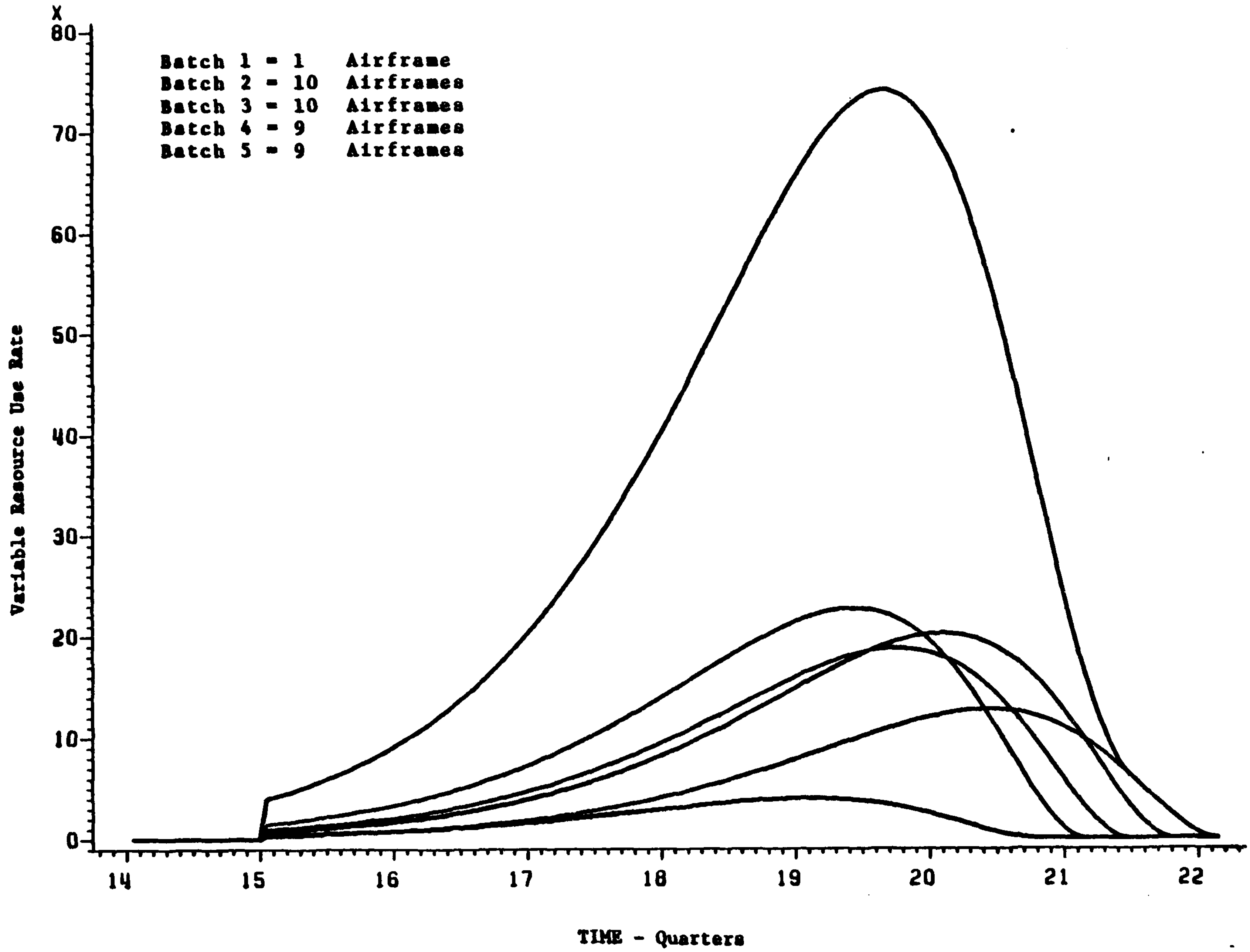

Figure 1. Simulated optimal time path of resource use rate for a given lot of airframes.

reduced. Labor requirements for the entire program may be depicted as the sum of the lot requirements curve.

All these time paths conform nicely to our understanding of the way labor requirements vary over time on a program. Therefore, we concluded that the model adequately captures the time dimension of the production problem.

A second sensitivity analysis model concerned the effect of a change in delivery schedule on program costs. To illustrate these changes we considered two alternatives to the delivery schedule for lot eleven of the C-141. Table III shows the three delivery schedules for the eleventh lot of C-141 airframes. The standard for comparison is schedule A which describes the schedule reported by Orsini. In Schedule B the start of work on lot eleven is delayed until the beginning of quarter 16, and in schedule C the schedule is compressed by combining batch four and batch five into one batch at t_{i4}. Each of the schedules assigns all of the aircraft in a batch to the middle of the month of delivery.

Figure 2 shows the time path of undiscounted cumulative labor hours for the three schedules. An enlargement of Figure 2 is presented in Figure 3. Notice that, in terms of undiscounted cost, a delay of one quarter in the beginning of the production period has a relatively minor impact on cost. However, when batches four and five are combined into a single batch, the cost savings are substantial. One implication of this simulation is that changes in the delivery schedule that occur near the end of the production period have a relatively larger impact on undiscounted cost.

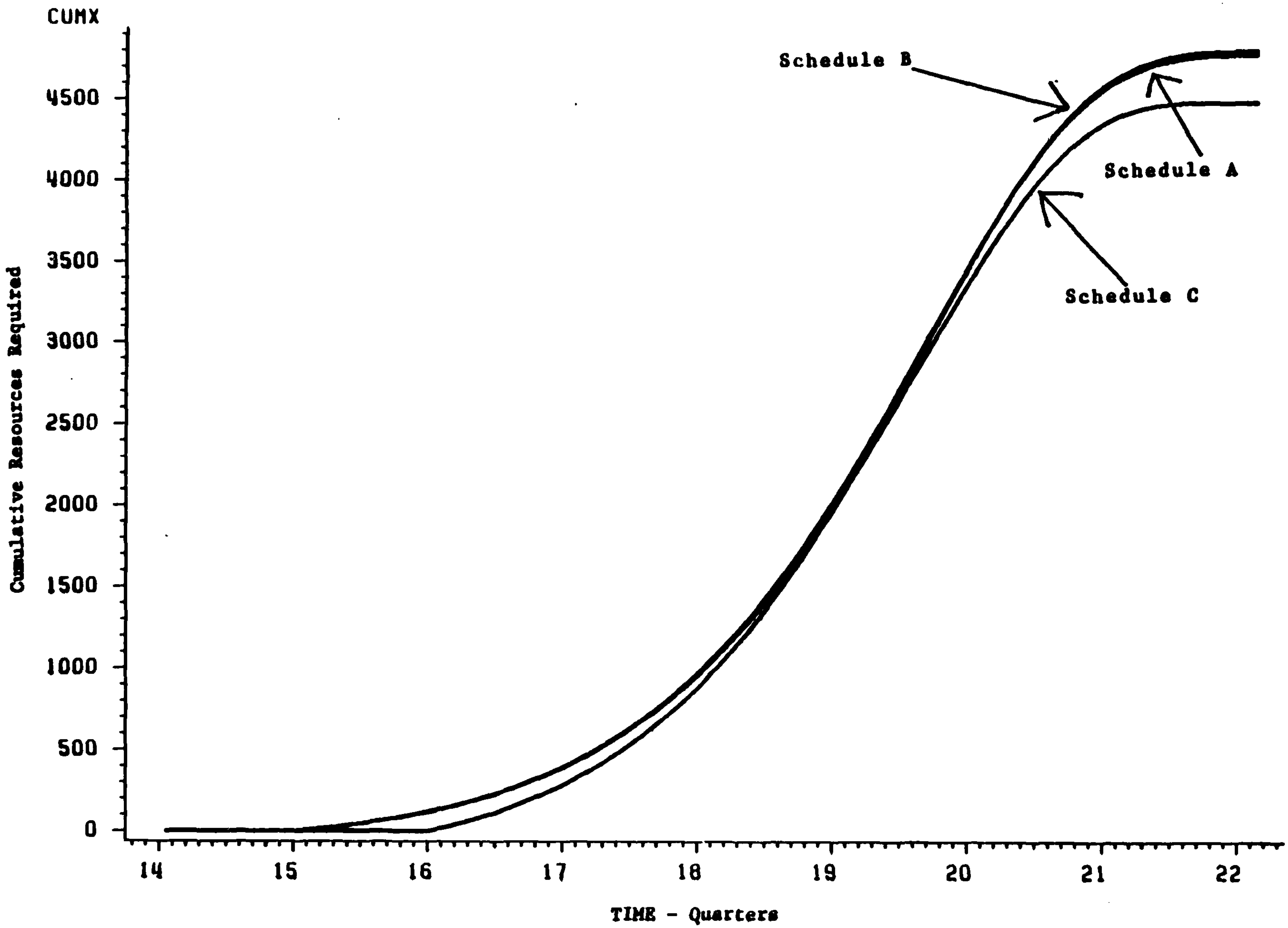

Figure 2. Simulated optimal time path of cumulative resource usage for three different delivery schedules

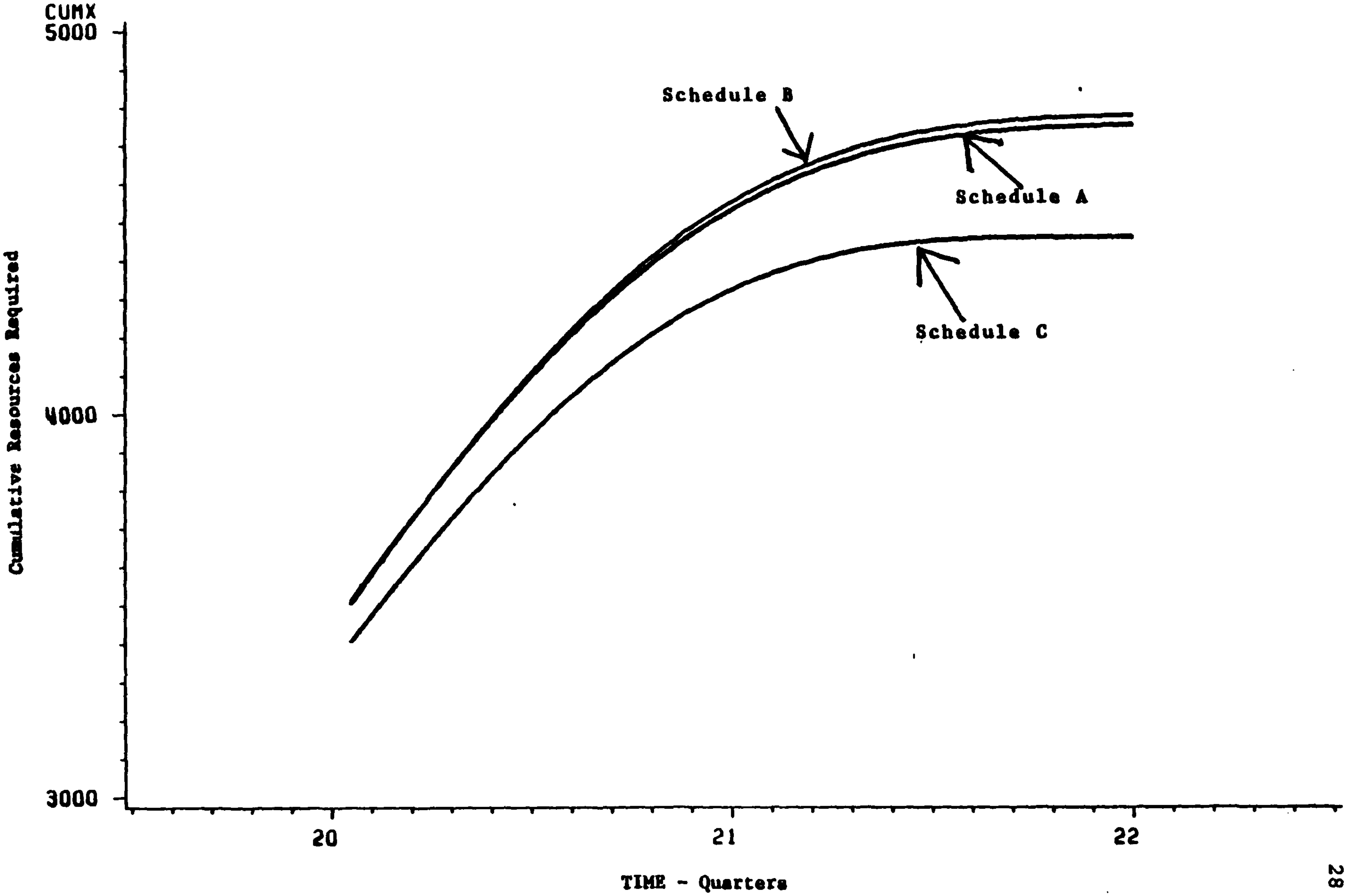

Figure 3. Simulated optimal time path of cumulative resource usage for three different delivery schedules (enlargement)

Table III

Alternative Delivery Schedule for Lot Eleven

Schedule	t_j*	t_{1j}	D_{1j}	t_{2j}	D_{2j}	t_{3j}	D_{3j}	t_{4j}	D_{4j}	t_{5j}	D_{5j}
A	15	20.83	1	21.17	10	21.5	8	21.83	9	22.17	5
B	16	20.83	1	21.17	10	21.5	8	21.83	9	22.17	5
C	15	20.83	1	21.17	10	21.5	8	21.83	14	22.17	0

*Time is measured in quarters.

This same delivery schedule simulation is further examined by looking at discounted cost. For any batch i in lot j total discounted program cost is given by

$$C'(t) = \int_{t_j}^{t} x_{ij}(\tau)\, e^{-\rho\tau} d\tau. \tag{2.28}$$

It follows from equation (2.19) that the optimal cost per batch is

$$C'(t) = K_5 \int_{t_j}^{t} (t_{ij}-\tau)^{\gamma\alpha/(\gamma-1)} e^{\rho\tau/(\gamma-1)} d\tau. \tag{2.29}$$

where

$$K_5 = A^{-\gamma} E^{-\delta\gamma} D\, (1-\varepsilon)^{-\gamma}$$

$$\Gamma^{-\gamma}[\rho(t_{ij}-t_j)/(\gamma-1), \alpha\gamma/(\gamma-1)+1]\; e^{\rho t_{ij}/(\gamma-1)}. \tag{2.30}$$

This integral, after appropriate transformation, may be expressed as an incomplete gamma function (see equation (2.12),) i.e.,

$$C'(t) = -K_5[\rho/(\gamma-1)]^{-(\gamma\alpha+\gamma-1)/(\gamma-1)} e^{\rho t_{ij}/(\gamma-1)}$$

$$\int_{\rho(t_{ij}-t_j)/(\gamma-1)}^{\rho(t_{ij}-t)/(\gamma-1)} u^{\gamma\alpha/(\gamma-1)} e^{-u} du. \qquad (2.31)$$

After integrating, the discounted cost function is found to be

$$C'(t) = A^{-\gamma} E^{-\delta\gamma} D^{\gamma(1-\varepsilon)} (1-\varepsilon)^{-\gamma} \Gamma^{-\gamma}[\rho(t_{ij}-t_j)/(\gamma-1), \alpha\gamma/(\gamma-1)+1]$$
$$[\rho/(\gamma-1)]^{(\gamma-1)(\alpha\gamma+\gamma-1)} \{\Gamma[\rho(t_{ij}-t_j)/(\gamma-1),$$
$$\alpha\gamma/(\gamma-1)+1] - \Gamma[\rho(t_{ij}-t)/(\gamma-1), \alpha\gamma/(\gamma-1)+1]\}. \qquad (2.32)$$

This function yields the discounted cumulative cost of producing one batch of airframes. Adding (2.32) over all the batches in a lot yields the discounted cost function for a lot. This function is estimated as

$$= \sum_{i=1}^{n_j} 0.631\ E_{ij}^{-0.283}\ D_{ij}^{0.758}\ \Gamma^{-1.041}[1.2(t_{ij}-t_j), 3.16] e^{-0.049 t_{ij}}$$
$$\{\Gamma[1.2(t_{ij}-t_j),\ 3.16] - \Gamma[1.2(t_{ij}-t),\ 3.16]\}. \qquad (2.33)$$

A simulation of the three delivery schedules using equation (2.33) is presented in Figure 4. When looking at discounted cost, the relative impact of changing delivery schedules is essentially the same. In terms of discounted cost, it is not very expensive to

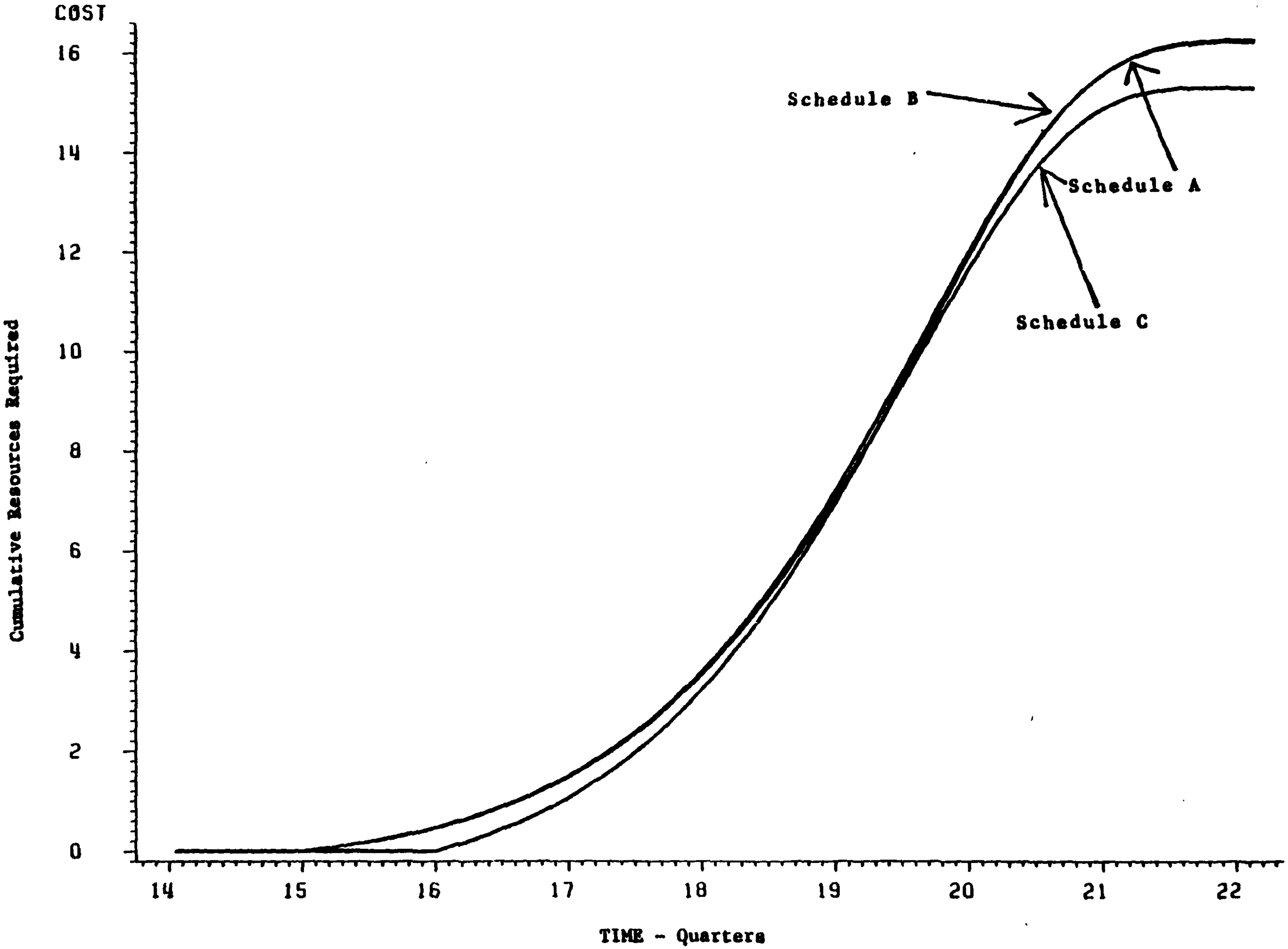

Figure 4. Simulated optimal time path of discounted total cost for three different delivery schedules

delay the production process, while the discounted cost impacts of combining the last two batches are considerable.

Even though these sensitivity analyses seemed to support Smith's conclusions, we found them disturbing. The analyses seem to indicate a savings to the Air Force merely by requiring the contractor to make deliveries in the same lot earlier than previously scheduled. (Schedule C) But, if the contractor is motivated to minimize cost, and if he can schedule production any time after the lot release date to meet the delivery schedule, then he could have achieved the same cost savings without the government action.

Of course it is possible that delivering early would create a break in production between lots. In the absence of further government action (advancing the release date for the next lot) the costs of the production break may outweigh the benefits of earlier delivery. This explanation for the result would imply a woeful lack of communication among the contractor, the Air Force and Congress. Because of this, we were concerned that the model was misspecified.

E. Conclusions: Four Effects

In reviewing these analyses we identified four means by which production scheduling might affect production efficiency and therefore cost. To aid our thinking about these effects, we use the concept of a production line to describe the four effects.

The first effect is the usual concept of learning. That is, over time, as each position on the production line becomes more experienced, the work at that position is performed more efficiently. This effect is modeled by including the term E_{ij}, the number of airframes delivered prior to the current batch, in equation (2.1).

The second effect is another concept of learning--learning may occur over time. Early in the program labor hours may be spent to learn how to produce more efficiently. Later in the program this may result in increased efficiency, independent of experience at a point on the production line. If this is the case, positions at the end of the production line work more efficiently on the same airframe than positions at the beginning of the line (the work at these positions may be performed as much as a year apart). This effect is modeled explicitly in (2.1). However, the terms $Q_{ij}(t)$ and $(t_{ij} - t)$ may capture this effect in addition to the fact that the nature of the work along the production line changes from beginning to end.

The third effect is due to increasing the speed of the production line. Unless there is a learning compensation, increasing the speed of the line is expected to require more labor at each position on the line. Furthermore, due to diminishing returns, the additional labor required is expected to be more than in proportion to the increase in speed. This effect is captured by the relation between $q_{ij}(t)$ and $X_{ij}(t)$ in (2.1).

The fourth effect of production scheduling on efficiency is due to crowding of the facilities as the production line is lengthened. One way to increase delivery rate is to increase the number of positions on the production line, reducing the amount of work to be done at each position, and increasing the total amount of work accomplished per unit of time. As facilities become crowded and tools and other fixed resources become overused, and this is expected to increase the unit cost of production. This last effect involves an interaction among batches of airframes in the facility at the same point in time. It is not captured by the present model, and this may very well account for the results observed in the sensitivity analyses above.

As a result of the sensitivity analyses, we conducted a thorough investigation of alternatives to the model focusing on effects two and four. In Chapter III one set of these alternatives is reported.

III. DYNAMIC MULTIPLE OUTPUT PRODUCTION FUNCTIONS

A. Introduction

This chapter centers on dynamic multiple output production functions and their use in constructing usable generalizations to the C-141 model. In approaching the problem in this way we attempt to model an additional dimension of the contractor's behavior. In addition to deciding about the time path of production rate, we attempt to model the contractor's decisions concerning the time path of resources devoted to learning how to produce (effect 2). We also model effects one and three explicitly. Effect four depends on particular characteristics of the production line and the delivery schedule. It is not included in the models in this chapter. There are many possible specifications for production functions with different learning hypotheses. The approach taken in Chapter II is to assume neutral technological change and augment learning as cumulative output increases. Another possibility is to treat learning as a separate output in a multiple output production function. This learning output increases the stock of knowledge which enters the production function in the next instance of time as an input. All the models in this chapter are based on the assumption that the contractor produces two products, airframes and knowledge, about how to produce airframes. They are all based on varieties of dynamic, multiple product production functions.

The theory of multiple output production functions is well defined for the static situation. However, very little is known about the dynamic properties of these functions. The strategy of this study is as follows:

1. Specify a cost minimization model using a Constant Elasticity of Transformation (CET) multiple output production function. Learning and output are produced using variable resources and the stock of knowledge as inputs. The analytical solution for this model is unknown.

2. The complexity of the CET model is reduced by looking at its most simple form, the Diewert model. An analytical solution to the cost minimizing Diewert model is found by making an assumption about the relationship between experience rate (learning) and output rate.

3. A dynamic multiple output cost minimizing model based on the Mundlak production function is specified. The analytical solution for this model is unknown.

4. A two production function model is specified, and the optimal time paths of learning and output are obtained by analytical methods.

Analytical solutions are always desirable for mathematical models, however, for dynamic models, the solutions are very difficult to obtain. In this study analytical solutions are presented when available, and furthermore, no attempt is made to present numerical solutions. The complexity of these models suggests that the numerical solutions be postponed for later research.

B. The CET Output Function

The Constant Elasticity of Transformation output function is a functional form that is similar to the CES production function. The CET function, which was first introduced by Powell and Gruen [24] has special restrictions on the parameters which insure that the transformation function has the proper convexity. The multiple output production function is

$$(\Sigma\alpha_i q_i^{\beta})^{1/\beta} = g(x) \qquad (3.1)$$

where g(x) is any appropriate input function, e.g., g(x) might be a Cobb-Douglas input function. The restrictions on the parameters require $\alpha_i > 0$ and $\beta > 1$. The restriction on β gives the CET function the appropriate convexity.

Consider the following specification which is relevant for this study. Two output variables are required, and the input function is assumed to be of the Cobb-Douglas type. The production function is

$$[\alpha_1 q^{\beta}(t)+\alpha_2 \ell^{\beta}(t)]^{1/\beta} = Ax^{1/\gamma}(t)L^{\delta}(t) \qquad (3.2)$$

where

$q(t)$ = output rate at time t,

$\ell(t)$ = experience rate at time t,

$x(t)$ = resource requirement rate at time t,

$L(t)$ = cumulative stock of knowledge at time t,

and

A = constant term.

The restrictions on α_i and β are as previously defined, γ is assumed to be greater than one, and δ is assumed to fall between zero and one. The assumption on γ implies decreasing returns to the variable factor, and the assumption on β is consistent with apriori knowledge about the learning curve.

The contractor's objective is to minimize production costs while satisfying the production function constraint and the boundary conditions. Since cost is measured in units of the variable resource, the objective may be stated as

$$\text{Min } C = {}_0\int^T x(t)e^{-\rho t}dt \qquad (3.3)$$

subject to:

$$[\alpha_1 q^\beta(t)+\alpha_2 \ell^\beta(t)]^{1/\beta} = Ax^{1/\gamma}(t)L^\delta(t), \qquad (3.4)$$

$$Q(0) = 0, \qquad (3.5)$$

$$Q(T) = V, \qquad (3.6)$$

$$L(0) = 0, \qquad (3.7)$$

and

$$L(T) = \text{free} \qquad (3.8)$$

where ρ is the discount rate. The initial stock of knowledge is assumed known while the final stock of knowledge is unknown. This assumption is for simplicity since in later specifications the value for the initial stock of knowledge will be estimated jointly with the other model parameters. The model as stated may be solved using Lagrange multipliers, but the solution procedure is simplified if the constraint (3.4) is absorbed into the objective functional (3.3). That is, solving (3.4) for x(t) yields the following resource requirement function:

$$x(t) = A^{-\gamma}L^{-\delta\gamma}(t)[\alpha_1 q^{\beta}(t)+\alpha_2 \ell^{\beta}(t)]^{\gamma/\beta}. \qquad (3.9)$$

Substitute (3.9) into (3.3) to obtain the transformed problem. The objective is

$$\text{Min } C = \int_0^T A^{-\gamma}L^{-\delta\gamma}(t)$$

$$[\alpha_1 q^{\beta}(t)+\alpha_2 \ell^{\beta}(t)]^{\gamma/\beta}e^{-\rho t}dt \qquad (3.10)$$

subject to:

$$Q(0) = 0, \qquad (3.5)$$

$$Q(T) = V, \qquad (3.6)$$

$$L(0) = 0, \qquad (3.7)$$

and

$$L(T) = \text{free}. \qquad (3.8)$$

The necessary conditions for a minimum require that the Lagrange-Euler equations be equated with zero. These conditions are

$$\gamma\alpha_1 A^{-\gamma} L^{-\delta\gamma}(t) q^{\beta-1}(t)$$
$$[\alpha_1 q^{\beta}(t)+\alpha_2 \ell^{\beta}(t)]^{\gamma/\beta-1} e^{-\rho t} = k, \quad (3.11)$$

and

$$-\delta\gamma A^{-\gamma} L^{-(\delta\gamma+1)}(t)[\alpha_1 q^{\beta}(t) +$$
$$\alpha_2 \ell^{\beta}(t)]^{\gamma/\beta} e^{-\rho t} - d/dt[\gamma\alpha_2 A^{-\gamma} L^{-\delta\gamma}(t)$$
$$\ell^{\beta-1}(t)[\alpha_1 q^{\beta}(t)+\alpha_2 \ell^{\beta}(t)]^{\gamma/\beta-1} e^{-\rho t}]=0. \quad (3.12)$$

This is a system of second order nonlinear differential equations for which the analytical solution is unknown. The difficulty involved in finding solutions to systems such as (3.11) and (3.12) suggests that a simpler specification is desirable. One such simplification, which is a special case of the CET output function, is the quadratic or Diewert output function.

C. The Diewert Output Function

One of the simplist multiple output production functions was first introduced by W.E. Diewert [6]. The general form of the multiple output production function is

$$(q'Bq)^{1/2} = g(x) \tag{3.13}$$

where q is an $n \times 1$ vector of output variables, B is an $n \times n$ symmetric positive definite matrix, and g(x) is an appropriate input function. As shown in (3.14) this function may be specified to include any number of outputs, but the requirements for this study suggest that the number be limited to two. To complete the specification, the input side is assumed to be of the multiplicative Cobb-Douglas type. The specification is

$$[q^2(t) + \ell^2(t)]^{1/2} = Ax^{1/\gamma}(t)L^{\delta}(t) \tag{3.15}$$

The solution to the cost minimization model yields a time path of minimum discounted program cost subject to the production function constraint, that is,

$$\text{Min } C = \int_0^T x(t)e^{-\rho t}dt \tag{3.3}$$

subject to:

$$q^2(t) + \ell^2(t) = A^2x^{2/\gamma}(t)L^{2\delta}(t), \tag{3.15}$$

$$Q(0) = 0, \tag{3.5}$$

$$Q(T) = V, \tag{3.6}$$

$$L(0) = 0, \tag{3.7}$$

and

$$L(T) = \text{free} \tag{3.8}$$

where the terminal time T and terminal cumulative output V are assumed known. The terminal stock of knowledge is assumed to be unknown.

The resource requirement function is found by solving (3.15) for the variable composite resource. The function is

$$x(t) = A^{-\gamma}L^{-\delta\gamma}(t)[q^2(t)+\ell^2(t)]^{\gamma/2}. \tag{3.16}$$

The problem is restated by substituting (3.16) into the objective functional (3.3), that is,

$$\text{Min } C = \int_0^T A^{-\gamma}L^{-\delta\gamma}(t)[q^2(t) + \ell^2(t)]^{\gamma/2}e^{-\rho t}dt. \tag{3.17}$$

The necessary conditions for an optimal solution require that the Lagrange-Euler equations be equated with zero. These conditions are stated as

$$\gamma A^{-\gamma} L^{-\delta\gamma}(t) q(t) [q^2(t) + \ell^2(t)]^{\gamma/2-1} e^{-\rho t} = k, \qquad (3.18)$$

and

$$-\delta\gamma A^{-\gamma} L^{-(\delta\gamma+1)}(t)[q^2(t)+\ell^2(t)]^{\gamma/2} e^{-\rho t} - d/dt[\gamma A^{-\gamma} L^{-\delta\gamma}(t)[q^2(t) + \ell^2(t)]^{\gamma/2-1} e^{-\rho t} = 0. \qquad (3.19)$$

This system of differential equations is second order and nonlinear, and at this point in time the analytical solution for Q(t) and L(t) is unknown. Furthermore, assuming a zero discount rate does not produce a more tractable problem.

Even if the solution is unknown, it is still instructive to note the importance of the solution. In this model experience rate is treated as a decision variable. Resources are diverted from production in order to produce learning, but this produced experience re-enters the production process as enhanced knowledge with the potential of reducing discounted cost at some later instance. The suggestion is that there is some optimal trade-off between learning and output rate.

The control format is particularly pertinent for solving the problem. There is always the question of how to measure learning and hence the stock of knowledge. For that matter, with respect to airframe production, there is always a question about how to measure production rate. The control formulation eliminates both of these problems since experience rate and production rate are "optimized" out of the problem, i.e., they are both decision variables. The solution yields discounted program cost as a function of time.

With this optimal expression for cost, numerous hypothetical policy simulations are possible. The cost impacts of primary interest include exogenous changes in production rate via changes in delivery schedules. This information would be particularly helpful in updating cost estimates during the production period of an airframe program.

D. The Restricted Model

Since the solution of the dynamic Diewert model is unknown, additional information is obtained by examining a restricted model. Suppose the model is stated with an additional constraint. The objective is

$$\text{Min } C = \int_0^T x(t)e^{-\rho t}dt \tag{3.3}$$

subject to:

$$q^2(t) + \ell^2(t) = A^2x^{2/\gamma}(t)L^{2\delta}(t), \tag{3.15}$$

$$L(t) = \alpha Q(t) + K \quad \text{or} \quad \ell(t) = \alpha q(t), \tag{3.20}$$

$$Q(0) = 0, \tag{3.5}$$

$$Q(T) = V, \tag{3.6}$$

$$L(0) = K, \tag{3.21}$$

and

$$L(T) = \alpha V + K. \tag{3.22}$$

Notice that the initial stock of knowledge for this model is assumed to be some constant K. This seems to be a reasonable assumption for the airframe industry. This definition, along with the constraint on L(t), defines the terminal condition on L(T). The problem is now expressed as a fixed endpoint problem. The additional constraint defines a specific relationship between experience rate and output rate, i.e.,

$$\ell(t) = \alpha q(t). \tag{3.20}$$

Graphically, the restriction is presented in figure .. The parameter α defines a particular location on the production possibility curve. Any expansion in output must occur along the ray with slope α. This restriction is stringent, but it permits solving the problem by using the calculus of variations.

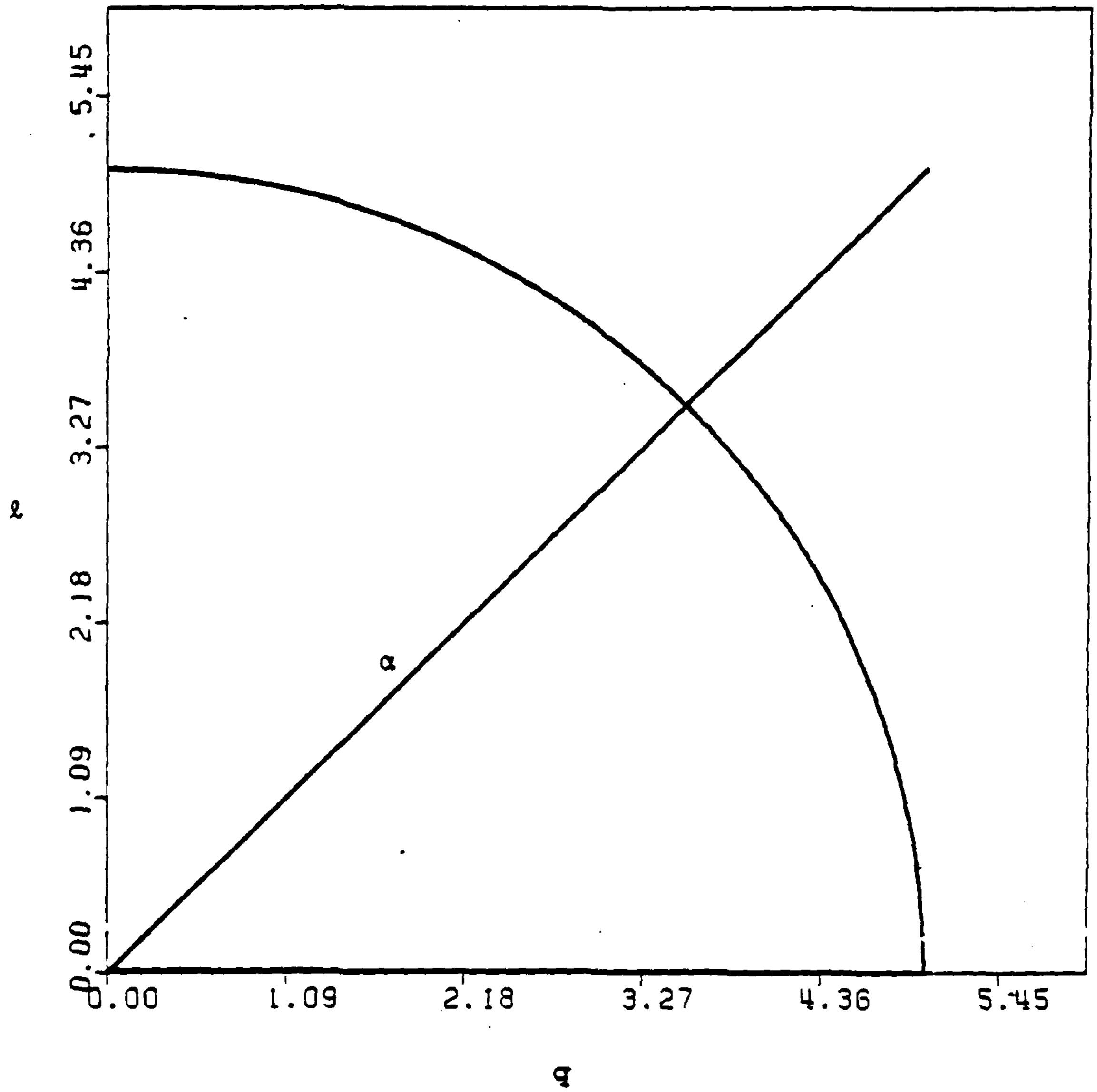

Figure 5. Production Possibility Curve

The solution is obtained by performing a series of transformations. First, substitute (3.20) into (3.15) and solve for x(t). This yields the following resource requirement function:

$$x(t) = A^{-\gamma}(1+\alpha^2)^{\gamma/2}q^{\gamma}(t)[\alpha Q(t)+K]^{-\delta\gamma}. \qquad (3.23)$$

Let

$$Z(t) = \alpha Q(t) + K. \qquad (3.24)$$

It follows that

$$z(t) = dZ/dt = \alpha q(t). \qquad (3.25)$$

The problem may now be stated as one in the new state variable Z(t). The objective is

$$\text{Min } C = \int_0^T A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}z^{\gamma}(t)Z^{-\delta\gamma}(t)e^{-\rho t}dt \qquad (3.26)$$

subject to:

$$Z(0) = K, \qquad (3.27)$$

and

$$Z(T) = \alpha V + K. \qquad (3.28)$$

The necessary condition for an optimum is that the Lagrange-Euler equation be equated with zero. This condition is stated as

$$\partial I/\partial Z - d/dt(\partial I/\partial z) = 0 \qquad (3.28)$$

where

$$I(z,Z,t)=A^{-\gamma}(1+\alpha^2)^{\gamma/2}K^{-\gamma}z^{\gamma}(t)Z^{-\delta\gamma}(t)e^{-\rho t}dt \qquad (3.29)$$

is the intermediate function for the problem.

This problem is still very difficult to solve since the differential equation (3.28) is nonlinear and second order. However, an additional transformation leads to a straightforward analytical solution. Let

$$Y(t) = Z^{1-\delta}(t)/(1-\delta). \qquad (3.30)$$

This implies that

$$y(t) = dY/dt = Z^{-\delta}(t)z(t). \qquad (3.31)$$

Substituting into (3.26) and redefining the boundary conditions leads to a third representation of the cost minimization model. The objective is

$$\text{Min } C={}_0\int^T A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}y^{\gamma}(t)e^{-\rho t}dt \qquad (3.32)$$

subject to:

$$Y(0) = K^{1-\delta}/(1-\delta), \tag{3.33}$$

and

$$Y(T) = (\alpha V+K)^{1-\delta}/(1-\delta). \tag{3.34}$$

Since the intermediate function for the transformed problem does not depend explicitly on Y(t), the Lagrange-Euler equation for this problem integrates to a constant, that is,

$$\gamma A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}y^{\gamma-1}(t)e^{-\rho t} = k_1. \tag{3.35}$$

The solution for optimal y(t) is

$$y(t) = k_2 e^{\rho t/(\gamma-1)} \tag{3.36}$$

where

$$k_2 = k_1^{1/(\gamma-1)}\gamma^{-1/(\gamma-1)}A^{\gamma/(\gamma-1)} (1+\alpha^2)^{-\gamma/[2(\gamma-1)]}. \tag{3.37}$$

It also follows from (3.36) that

$$Y(t) = k_2 \int_0^t e^{\rho t/(\gamma-1)}dt + k_3. \tag{3.38}$$

After integrating (3.38) and imposing the boundary conditions (3.33) and (3.34), the following expression is found for optimal Y(t).

$$Y(t)=[e^{\rho T/(\gamma-1)}-1]^{-1}[e^{\rho t/(\gamma-1)}-1] [(\alpha V+K)^{1-\delta}/(1-\delta)-K^{1-\delta}/(1-\delta)] + K^{1-\delta}/(1-\delta). \tag{3.39}$$

Also, since $Y(t)=Z^{1-\delta}(t)/(1-\delta)$, optimal Z(t) is

$$Z(t)=\{[e^{\rho T/(\gamma-1)}-1]^{-1}[e^{\rho t/(\gamma-1)}-1] [(\alpha V+K)^{1-\delta}-K^{1-\delta}]+K^{1-\delta}\}^{1/(1-\delta)}. \tag{3.40}$$

By applying (3.24) to (3.40) the optimal time path for cumulative output is obtained. The optimal time path is

$$Q(t)=\alpha^{-1}\{[e^{\rho T/(\gamma-1)}-1]^{-1}[e^{\rho t/(\gamma-1)}-1] [(\alpha V+K)^{1-\delta}-K^{1-\delta}]+K^{1-\delta}\}^{1/(1-\delta)}-K\alpha^{-1}. \tag{3.41}$$

Also, differentiation of (3.41) yields the following expression for optimal production rate:

$$q(t)=\alpha^{-1}\{\rho/[(1-\delta)(\gamma-1)]\}[(\alpha V+K)^{1-\delta}-K^{1-\delta}] [e^{\rho T/(\gamma-1)}-1]^{-1}\{[e^{\rho T/(\gamma-1)}-1]^{-1} [e^{\rho t/(\gamma-1)}-1]\ [(\alpha V+K)^{1-\delta}-K^{1-\delta}] +K^{1-\delta}\}^{\delta/(1-\delta)}e^{\rho t/(\gamma-1)}. \tag{3.42}$$

Substitution of (3.41) and (3.42) into (3.23) yields the optimal resource requirement function. The resources required at time t are

$$x(t)=A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}\{\rho/[(1-\delta)(\gamma-1)]\}^{\gamma} K_4{}^{\gamma}e^{\rho\gamma t/(\gamma-1)} \qquad (3.43)$$

where

$$K_4 = [e^{\rho T/(\gamma-1)}-1]^{-1}[(\alpha V+K)^{1-\delta}-K^{1-\delta}]. \qquad (3.44)$$

The optimal resource requirement function is inserted into the objective functional to obtain program cost as a function of time. Using (3.3), the relevant integral is

$$C(t) = \int_0^t A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma} \{\rho/[(1-\delta)(\gamma-1)]\}^{\gamma}K_4{}^{\gamma}e^{\rho\tau/(\gamma-1)}d\tau. \qquad (3.45)$$

After performing the integration, the optimal expression for total discounted program cost is

$$C(t) = A^{-\gamma}(1+\alpha^2)^{\gamma/2}\alpha^{-\gamma}\{\rho/[(1-\delta)(\gamma-1)]\}^{\gamma} [e^{\rho/(\gamma-1)}-1]^{-\gamma}[(\alpha V+K)^{1-\delta}-K^{1-\delta}]^{\gamma} [(\gamma-1)/\rho][e^{\rho t/(\gamma-1)}-1]. \qquad (3.46)$$

It is important to note that (3.46) is a function of time and the parameters of the model. This completely eliminates the problem of having to determine appropriate measurements for $q(t)$ and $\ell(t)$. These variables are removed from the problem via the optimization procedure.

After examining the restricted model closely, the following becomes evident. Although the model may produce interesting results, it will not necessarily yield the trade-offs between production and learning that are available with the Diewert model. Still, the model is somewhat unique in that it models the situation where there is an available initial stock of knowledge, but conceptually it adds very little to the theory of multiple output production functions. The addition of the restriction (3.20) alters the model in such a way that the multiple output technology is transformed into a single output technology. This is seen by examining the resource requirement function (3.23), i.e., notice that the single input x is a function of the single output q. In essence, this is the problem that was solved by Womer [38].

However, there is still some useful information to be gained from this model. The model is capable of exploring the impact on discounted cost of not only exogenous changes in production rates and delivery schedules (changes in V and T), but also changes in α. Since α determines the proportion of resources that are diverted to learning, the model can examine the relative impact on discounted cost of a change in this proportion.

Intuition suggests that, at least initially, total discounted cost will rise since there is a cost associated with learning. However, looking at the total cost function (3.47), the implications of changing α are not clear, since the impact on cost depends on the magnitudes of the parameters. The cost impact of a change in α is an econometric problem that can only be resolved empirically.

E. <u>The Mundlak Output Function</u>

Mundlak [18] presents the theoretical properties of a transcendental multiple output production function that is a generalization of the Cobb-Douglas production function. The production function may be stated as

$$q^{\omega}(t)\ell^{\xi}(t)e^{\beta_1 q(t)+\beta_2 \ell(t)} = Ax^{1/\gamma}(t)L^{\delta}(t)e^{\beta_3 x(t)+\beta_4 L(t)}. \quad (3.47)$$

The first and second order conditions for profit maximization place the following restrictions on the parameters: $\omega<0$, $\xi<0$, $\beta_1>|\omega/q|$, $\beta_2>|\xi/\ell|$, $1/\gamma>0$, $\delta>0$, $\beta_3>-1/(\gamma x)$; and $\beta_4>-\delta/L$. The restricions on the β_i's on the output side are particularly important since they preclude the existence of a Cobb-Douglas output function. Also, the nature of this particular problem suggests two additional restrictions on the parameters. The scale parameter γ is assumed to be greater than one, and the learning parameter δ is assumed to fall between zero and one.

The contractor's objective is to minimize the cost of producing V units by time T while satisfying the production function side relation. The objective may be stated as

$$\text{Min } C = \int_0^T x(t)e^{-\rho t}dt \qquad (3.3)$$

subject to:

$$q^{\omega}(t)\ell^{\xi}(t)e^{\beta_1 q(t)+\beta_2 \ell(t)} = Ax^{1/\gamma}(t)L^{\delta}(t)e^{\beta_3 x(t)+\beta_4 L(t)}, \qquad (3.47)$$

$$Q(0) = 0, \qquad (3.5)$$

$$Q(T) = V, \qquad (3.6)$$

$$L(0) = 0, \qquad (3.7)$$

and

$$L(T) = \text{free}. \qquad (3.8)$$

This problem is very difficult to solve by classical variational techniques. The procedure used on the previously defined problems involved solving the production function for the variable resource and substituting the constraint directly into the objective functional. That strategy clearly will not work with the above specification. One way to alleviate this problem is to assume that the input function is a Cobb-Douglas production function. Given this assumption (3.47) reduces to

$$q^{\omega}(t)\ell^{\xi}(t)e^{\beta_1 q(t)+\beta_2 L(t)} = Ax^{1/\gamma}(t)L^{\delta}(t). \quad (3.48)$$

The resource requirement function is obtained by solving (3.48) for the variable resource x(t). The function is

$$x(t)=A^{-\gamma}q^{\omega\gamma}(t)\ell^{\xi\gamma}(t)L^{-\delta\gamma}(t)$$
$$e^{\beta_1\gamma q(t)+\beta_2\gamma\ell(t)}. \quad (3.49)$$

The transformed cost minimization problem is now stated as

$$\text{Min } C = {}_0\int^T A^{-\gamma}q^{\omega\gamma}(t)\ell^{\xi\gamma}(t)L^{-\delta\gamma}(t)$$
$$e^{\beta_1\gamma q(t)+\beta_2\gamma\ell(t)-\rho t}dt \quad (3.50)$$

subject to:

$$Q(0) = 0, \quad (3.5)$$

$$Q(T) = V, \quad (3.6)$$

$$L(0) = 0, \quad (3.7)$$

and

$L(T) = \text{free}.$ (3.8)

Notice that this problem is much easier to solve if β_2 is assumed to be zero. If this were the case the problem could be transformed in such a way that both Lagrange-Euler equations could be integrated directly with respect to time. That is, let $\beta_2=0$ and define $Z(t) = L^{\delta/\xi+1}(t)/(\delta/\xi+1)$. This implies that $z(t) = dZ/dt = L^{\delta/\xi}(t)\ell(t)$. With this change of variables the objective functional depends only on $q(t)$ and $z(t)$. Since $Z(t)$ and $Q(t)$ do not appear in the formulation, the problem is much easier to solve. Unfortunately it is impossible to make the described substitution since $\beta_2=0$ violates the restrictions on the parameter values.

The necessary conditions for an extremal require that the Lagrange-Euler equations be equated with zero, i.e.,

$$\partial I/\partial Q - d/dt[\partial I/\partial q] = 0, \qquad (3.51)$$

$$\partial I/\partial L - d/dt[\partial I/\partial \ell] = 0 \qquad (3.52)$$

where I is the integrand of (3.50). After taking the appropriate derivatives the necessary conditions are stated as

$$[\omega\gamma q^{\omega\gamma-1}(t)+\beta_1\gamma q^{\omega\gamma+1}(t)]A^{-\gamma}\ell^{\xi\gamma}(t) \\ L^{-\delta\gamma}(t)e^{\beta_1\gamma q(t)+\beta_2\gamma\ell(t)-\rho t} = k, \qquad (3.53)$$

and

$$-\delta A^{-\gamma}q^{\omega\gamma}(t)\ell^{\xi\gamma}(t)L^{-(\delta\gamma+1)}(t)$$
$$e^{\beta_1\gamma q(t)+\beta_2\gamma\ell(t)-\rho t} - d/dt\{[\xi\gamma\ell^{\xi\gamma-1}(t)$$
$$+ \beta_2\gamma\ell^{\xi\gamma}(t)]A^{-\gamma}q^{\omega\gamma}(t)L^{-\delta\gamma}(t)$$
$$e^{\beta_1\gamma q(t)+\beta_2\gamma\ell(t)-\rho t}\} = 0. \qquad (3.54)$$

This system of differential equations is nonlinear and second order, and at this point in time, the solution for Q(t) and L(t) is unknown. The degree of difficulty associated with obtaining an analytical solution to this system suggests that another model specification is desirable.

F. The Two Production Function Model

Consider the case where learning and output are produced by two separate production technologies. One possible specification is two Cobb-Douglas production functions, that is,

$$q(t) = a_1 x_q^{1/\gamma}(t)L^{\alpha}(t), \qquad (3.55)$$

and

$$\ell(t) = a_2 x_\ell^{1/\beta}(t)L^{\delta}(t). \qquad (3.56)$$

With this specification the use rate of the composite resource is segregated into two parts, that allocated to output $x_q(t)$ and that allocated to learning $x_\ell(t)$. These inputs, combined with the cumulative stock of knowledge

L(t), are used to produce two products, output q(t) and learning $\ell(t)$. The following assumptions define the admissible ranges for the parameters: $0 \leq \alpha \leq 1$, $0 \leq \delta \leq 1$, $\gamma \geq 1$, and $\beta \geq 1$.

The objective of the firm is to minimize its cost of production subject to the production function constraints. This may be stated as

$$\text{Min } C = \int_0^T [x_q(t) + x_\ell(t)]dt \tag{3.57}$$

subject to:

$$q(t) = a_1 x_q^{1/\gamma}(t) L^{\alpha}(t), \tag{3.55}$$

$$\ell(t) = a_2 x_\ell^{1/\beta}(t) L^{\delta}(t), \tag{3.56}$$

$$Q(0) = 0, \tag{3.5}$$

$$Q(T) = V, \tag{3.6}$$

$$L(0) = M, \tag{3.58}$$

and

$$L(T) = \text{free}. \tag{3.8}$$

Notice that in this model cost is not discounted. From a mathematical point of view this simplifies the solution considerably, and from an economic point of view this assumption is often appropriate. That is, many contracts state that the contractor receive full cost recovery throughout the life of the contract. Under these conditions discounting may not be a major concern. The contractor doesn't care if the cost is incurred now or later.

The solution of (3.55) and (3.56) for $x_q(t)$ and $x_\ell(t)$ yields the following resource requirement functions:

$$x_q(t) = q^{\gamma}(t)a_1^{-\gamma}L^{-\alpha\gamma}(t). \tag{3.59}$$

and

$$x_\ell(t) = \ell^{\beta}(t)a_2^{-\beta}L^{-\delta\beta}(t). \tag{3.60}$$

Substituting the resource requirement functions into the objective functional eliminates the production function constraints. The objective functional is now stated as

$$\text{Min } C = \int_0^T [q^{\gamma}(t)a_1^{-\gamma}L^{-\alpha\gamma}(t) + \ell^{\beta}(t)a_2^{-\beta}L^{-\delta\beta}(t)]dt. \tag{3.61}$$

This problem may be solved by the usual methods, but a transformation simplifies the solution procedure. Let

$$Z(t) = L^{1-\delta}(t)/(1-\delta). \tag{3.62}$$

This implies that

$$L(t) = Z^{1/(1-\delta)}(t)(1-\delta)^{1/(1-\delta)}. \tag{3.63}$$

Also, the differentiation of (3.62) yields the following relationship:

$$z(t) = dZ/dt = L^{-\delta}(t)\ell(t). \tag{3.64}$$

After making the appropriate substitutions the transformed problem is

$$\text{Min } C = {}_0\int^T [q^{\gamma}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)} (1-\delta)^{-\alpha\gamma/(1-\delta)} + z^{\beta}(t)a_2^{-\beta}]dt \tag{3.65}$$

subject to:

$$Q(0) = 0, \tag{3.5}$$

$$Q(T) = V, \tag{3.6}$$

$$Z(0) = M^{1-\delta}/(1-\delta), \tag{3.66}$$

and

$$Z(T) = \text{free}. \tag{3.67}$$

An equivalent way to present the above problem is as a problem in optimal control theory. The objective is stated as

$$\text{Min } C = \int_0^T [u_1^{\gamma}(t) a_1^{-\gamma} Z^{-\alpha\gamma/(1-\delta)} (1-\delta)^{-\alpha\gamma/(1-\delta)} + u_2^{\beta}(t) a_2^{-\beta}] dt \tag{3.68}$$

subject to:

$$q(t) = u_1(t), \tag{3.69}$$

$$z(t) = u_2(t), \tag{3.70}$$

$$Q(0) = 0, \tag{3.5}$$

$$Q(T) = V, \tag{3.6}$$

$$Z(0) = M^{1-\delta}/(1-\delta), \tag{3.66}$$

and

$$Z(T) = \text{free}. \tag{3.67}$$

The control variables for the problem, $u_1(t)$ and $u_2(t)$, are the time rates of change of the state variables, i.e, $u_1(t)$

= q(t) and $u_2(t) = z(t)$. The Hamiltonian function for the problem is

$$H = u_1^{\gamma}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(t)(1-\delta)^{-\alpha\gamma/(1-\delta)}+$$

$$u_2^{\beta}(t)a_2^{-\beta}+\lambda_1(t)u_1(t)+\lambda_2(t)u_2(t). \quad (3.71)$$

The necessary conditions for defining an extremal require that the equations of motion, the adjoint conditions, and the Hamiltonian conditions hold simultaneously. The equations of motion are

$$\partial H/\partial\lambda_1 = q(t) = u_1(t), \quad (3.72)$$

$$\partial H/\partial\lambda_2 = z(t) = u_2(t). \quad (3.73)$$

The adjoint conditions are

$$d\lambda_1/dt = -\partial H/\partial Q = 0, \quad (3.74)$$

$$d\lambda_2/dt = -\partial H/\partial Z = [\alpha\gamma/(1-\delta)]u_1^{\gamma}(t)a_1^{-\gamma}$$

$$Z^{(\delta-\alpha\gamma-1)}(t)(1-\delta)^{-\alpha\gamma/(1-\delta)}. \quad (3.75)$$

The Hamiltonian conditions are

$$\partial H/\partial u_1 = \gamma u_1^{\gamma-1}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(t)$$

$$(1-\delta)^{-\alpha\gamma/(1-\delta)} + \lambda_1(t) = 0, \quad (3.76)$$

$$\partial H/\partial u_2 = \beta u_2^{\beta-1}(t)a_2^{-\beta} + \lambda_2(t) = 0. \qquad (3.77)$$

The simultaneous solution of these conditions requires solving two second order nonlinear differential equations. This implies that there are four constants to be determined by the boundary conditions. Three of the constants are determined by the given boundary conditions, and the fourth is given by the natural boundary condition, that is, since Z(T) is free, the condition z(T)=0 determines the fourth constant. The algebra of the solution procedure is as follows.

Substituting (3.72) into (3.77), solving for $\lambda_2(t)$, and differentiating with respect to t yields the following expression for $d\lambda_2/dt$:

$$d\lambda_2/dt = -\beta(\beta-1)z^{\beta-2}a_2^{-\beta}(d^2Z/dt^2). \qquad (3.78)$$

Equate (3.78) with (3.75) to yield the first Lagrange-Euler equation. The expression is

$$\begin{aligned}&\beta(\beta-1)z^{\beta-2}(t)a_2^{-\beta}(d^2Z/dt^2) + [\alpha\gamma/(1-\delta)]q^{\gamma}(t)\\&\quad a_1^{-\gamma}Z^{(\delta-\alpha\gamma-1)/(1-\delta)}(t)\\&\quad (1-\delta)^{-\alpha\gamma/(1-\delta)}=0. \qquad (3.79)\end{aligned}$$

To obtain the second Lagrange-Euler equation, make use of (3.72), (3.74), and (3.76). Substitute (3.72) into (3.76) to eliminate the control variable. Differentiate (3.76) with respect to time and equate with (3.74) to obtain the desired result. The result is stated as

$$d\lambda_1/dt = d/dt[-\gamma q^{\gamma-1}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(t)$$
$$(1-\delta)^{-\alpha\gamma/(1-\delta)}] = 0. \qquad (3.80)$$

This equation may be integrated directly with respect to time, i.e.,

$$-\gamma q^{\gamma-1}(t)a_1^{-\gamma}Z^{-\alpha\gamma/(1-\delta)}(t)$$
$$(1-\delta)^{-\alpha\gamma/(1-\delta)} = k'_1. \qquad (3.81)$$

It is possible to solve (3.81) for q(t) and state the result in compact notation as

$$q(t) = k_1 Z^{\eta}(t) \qquad (3.82)$$

where $\eta=\alpha\gamma/[(1-\delta)(\gamma-1)]$. This expression for q(t) is substituted into (3.79) and the number of equations is reduced by one. The single necessary condition is stated as

$$d^2Z/dt^2 = \beta^{-1}(\beta-1)^{-1}z^{2-\beta}(t)a_2^{\beta}[\alpha\gamma/(1-\delta)]$$
$$Z^{(1+\alpha\gamma+\delta\gamma-\gamma-\delta)/[(\gamma-1)(1-\delta)]}(t)$$
$$k_1^{\gamma}(1-\delta)^{\eta}. \qquad (3.83)$$

This is a second order nonlinear differential equation which may be stated in compact notation as

$$d^2Z/dt^2 = Az^{2-\beta}(t)Z^{\eta-1}(t) \qquad (3.84)$$

where the constant term is

$$A = \beta^{-1}(\beta-1)^{-1}a_2{}^{\beta}[\alpha\gamma/(1-\delta)]k_1{}^{\gamma}(1-\delta)^{\eta}, \qquad (3.85)$$

and

$$\eta-1 = (1+\alpha\gamma+\delta\gamma-\gamma-\delta)/[(\gamma-1)(1-\delta)]. \qquad (3.86)$$

The solution procedure begins by transforming the problem to achieve a reduction in order. Let P(t) = z(t), and the following is true:

$$dP/dt = (dP/dZ)(dZ/dt) = p(t)z(t). \qquad (3.87)$$

Therefore,

$$p(t)P(t) = d^2Z/dt^2 = AZ^{\eta-1}(t)P^{2-\beta}(t) \qquad (3.88)$$

or

$$p(t)P^{\beta-1}(t) = AZ^{\eta-1}(t). \qquad (3.89)$$

Now, consider the following transformation. Let

$$H(t) = P^{\beta}(t)/\beta. \qquad (3.90)$$

This implies that

$$h(t) = dH/dt = P^{\beta-1}(t)p(t). \qquad (3.91)$$

Now, substitute directly into (3.89) to obtain

$$h(t) = AZ^{\eta-1}(t). \qquad (3.92)$$

It follows from the above that

$$\int dH(t) = \int AZ^{\eta-1}(t)dZ. \qquad (3.93)$$

Integrate and substitute (3.181) for H(t) to obtain

$$P^{\beta}(t)/\beta = AZ^{\eta}(t)/\eta + k'_2 \qquad (3\ 94)$$

where k'_2 is a constant of integration. Since P(t) = z(t), the above may be simplified further, that is,

$$dZ/dt = [A\beta Z^{\eta}(t)/\eta + k_2]^{1/\beta}. \qquad (3.95)$$

This function may be inverted, leaving t as the following function of Z(t):

$$\int_{M^{1-\delta}/(1-\delta)}^{Z(t)} [A\beta Z^{\eta}/\eta + k_2]^{-1/\beta} dZ. \qquad (3.96)$$

Recall that an expression is needed that gives Z as a function of t. This allows us to apply the boundary conditions on Z(t), determine the constants, and define one of the extremals. Since (3.82) relates Z(t) to q(t), the boundary conditions on Q(t) can be used to and define the second extremal. Unfortunately, this procedure is easily stated but not easily executed.

The next step in the solution procedure is just a restatement of (3.96) in a slightly different form. After some algebraic manipulation (3.96) may be written as

$$t = k_2^{-1/\beta} \int_{M^{1-\delta}/(1-\delta)}^{Z(t)} [1+A\beta Z^{\eta}/\eta k_2]^{-1/\beta} dZ + k_3. \qquad (3.97)$$

The value of k_2 is determined by returning to (3.94). Since P(t)=z(t), (3.94) implies that

$$z^{\beta}(t) = A\beta Z^{\eta}(t)/\eta + k_2. \qquad (3.98)$$

Applying the natural boundary condition z(t) = 0, the resulting expression for k_2 is

$$k_2 = -A\beta Z^{\eta}(T)/\eta, \tag{3.99}$$

and (3.97) may be written as

$$t = [-A\beta Z^{\eta}(T)/\eta]^{-1/\beta} \int_{M^{1-\delta}/(1-\delta)}^{Z(t)} [1-Z^{-\eta}(T)Z^{\eta}]^{-1/\beta}\, dZ + k_3. \tag{3.100}$$

This integral appears uninviting, but a change of variables leads to a solution. Let

$$y = Z^{-\eta}(T)Z^{\eta}. \tag{3.101}$$

The integral may now be restated as

$$t = [-A\beta Z^{\eta}(T)/\eta]^{-1/\beta} Z(T)\eta^{-1} \int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{Z^{-\eta}(T)Z^{\eta}(t)} (1-y)^{(1-1/\beta)-1} y^{1/\eta-1} dy + k_3 \tag{3.102}$$

which is a form of the incomplete beta function.

The solution is now complete except for the determination of the constants of integration. At this point the value of k_3 is unknown, and there is also an unknown constant, k_1, in the A term. To determine these constants, first notice that (3.66) implies that $k_3=0$. The determination of the second constant requires a little more effort.

The strategy is to find an expression for $[-A\beta Z^{-\eta}(T)/\eta]$ that is in terms of known constants. The solution is as follows. Equation (3.95) may be written as

$$dt = [-A\beta Z^{-\eta}(T)/\eta]^{-1/\beta} [1-Z^{-\eta}(T)Z^{\eta}(t)]^{-1/\beta}dZ. \tag{3.103}$$

It also follows from (3.82) that

$$Q(\tau) = k_1 \int_0^t Z(\tau)d\tau. \tag{3.104}$$

After evaluating (3.103) at τ and changing variables in (3.104), it is possible to write Q as a function of Z.

$$Q(Z) = k_1[-A\beta Z^{\eta}(T)/\eta]^{-1/\beta} \int_{M^{1-\delta}/(1-\delta)}^{Z(t)} Z^{\eta}[1-Z^{-\eta}(T)Z^{\eta}]^{-1/\beta}dZ. \tag{3.105}$$

This integral results in an expression that is suitable for applying the boundary conditions on Q(t). In other words, (3.104) is transformed into an expression that is integrated with respect to Z instead of τ. Continuing with the solution, let

$$R = k_1[-A\beta Z^{\eta}(T)/\eta]^{-1/\beta}, \tag{3.106}$$

and

$$y = Z^{-\eta}(T)Z^{\eta}. \qquad (3.107)$$

Equation (3.105) may now be stated as a form of the incomplete Beta function. The appropriate integral is

$$Q(y) = RZ^{\eta+1}(T)\eta^{-1} \int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{Z^{-\eta}(T)Z^{\eta}(t)} y^{(1/\eta+1)-1}(1-y)^{(1-1/\beta)-1}dy + k_4. \qquad (3.108)$$

It is now possible to apply the boundary conditions on Q. The initial condition, $Q(0) = 0$, implies that $k_4 = 0$. The final condition, $Q(T) = V$, implies

$$R = VZ^{-1-\eta}(T)\eta\left\{\int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{1} y^{(1/\eta+1)-1}(1-y)^{(1-1/\beta)-1}dy\right\}^{-1} \qquad (3.109)$$

which is another form of the incomplete Beta function.

Since all of the integration constants are known, it is possible to state an expression that links optimal Q with optimal Z. The expression is

$$Q(t) = V\left\{\int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{1} y^{(1/\eta+1)-1}(1-y)^{(1-1/\beta)-1}dy\right\}^{-1}$$

$$\int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{Z^{-\eta}(T)Z^{\eta}(t)} y^{(1/\eta+1)-1}(1-y)^{(1-1/\beta)-1}dy. \qquad (3.110)$$

However, (3.110) is not the important result that follows from the solution for R. Recall that the objective is to determine the integration constants, and knowing R allows us to solve for $[-A\beta Z^{\eta}(T)/\eta]^{-1/\beta}$. This expression contains the unknown integration constant k_1. To determine the final constant solve (3.85) for k_1 and substitute into (3.106). After some algebraic manipulation, the desired result is

$$[-A\beta Z^{\eta}(T)/\eta]^{-1/\beta} = Z^{-\eta/(\beta-\gamma)}(T)\eta^{1/(\beta-\gamma)} R^{-\gamma/(\beta-\gamma)}(\beta-1)^{1/(\beta-\gamma)}a_2^{-\beta/(\beta-\gamma)} [\alpha\gamma/(1-\delta)]^{-1/(\beta-\gamma)}(1-\delta)^{-\eta/(\beta-\gamma)}. \tag{3.111}$$

Substitute (3.111) into (3.102) and to yield the optimal time path for Z(t). The expression is

$$t = Z^{(-\eta+\beta-\gamma)/(\beta-\gamma)}(T)\eta^{(1-\beta+\gamma)/(\beta-\gamma)} R^{-\gamma/(\beta-\gamma)}(\beta-1)^{1/(\beta-\gamma)}a_2^{-\beta/(\beta-\gamma)} [\alpha\gamma/(1-\delta)]^{-1/(\beta-\gamma)}(1-\delta)^{-\eta/(\beta-\gamma)} \int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{Z^{-\eta}(T)Z^{\eta}(t)} (1-y)^{(1-1/\beta)-1}y^{1/\eta-1}dy \tag{3.112}$$

where R is defined by (3.109).

The solution is complete. By using the inverse of the incomplete Beta function in (3.112), it is possible to determine optimal Z(t) for any t. Optimal Z(t) determines optimal L(t) via (3.63), and optimal Z(t) determines optimal Q(t) via (3.104).

IV. THE F-102 PROGRAM AND THE TWO PRODUCTION FUNCTION MODEL

A. The F102 Airframe Program

The F102 is a single-seat, supersonic, delta wing, all weather figher interceptor, and the TF102 is a two seated trainer version of the F102A. The F102 program was the overall responsibility of General Dynamics-CONVAIR with assistance from General Dynamics-Fort Worth. The support from Fort Worth was mainly on the TF102 nose and miscellaneous components.

The "F102 Program Cost History" [8] is a comprehensive document that includes numerous cost breakdowns by individual airframe on both the F102A and the TF102. These cost breakdowns are supplemented with monthly delivery schedules to provide the data for this study.

B. The Data

The F102 program was comprised of 1000 aircraft that were constructed during the years 1953 through 1958. Of these 1000, 889 are F102A interceptors and 111 are TF102 trainers. The variable of primary interest in the data base is direct labor hours for each airframe. Data is available for all 1000 airframes, but not all of the data is in the proper form to be used with the two production function model specified in Chapter III. Care has been taken to resolve all data problems, and the data is reorganized so that it is compatible with the previously specified theoretical model. The complete data base is presented in Appendix B.

One problem with this data is the cost differential between the F102 and the TF102 airframes. The largest part of this difference is caused by the additional nose cost for the TF102. If both models are included in the analysis, some data adjustment is required to compensate for the cost difference. One possible adjustment would be to delete the TF102 observations and complete the analysis using the 889 F102 observations. This procedure is not desirable since the learning on both of the airframes contributes to the cost behavior of each of the airframes. A more appropriate adjustment is the deletion of the additional nose cost from each of the TF102 airframes. For the TF102, the two-seat fuselage components were constructed in Fort Worth and shipped to San Diego for final assembly. The basic difference in the hours required on both models is due to the additional hours at Fort Worth. (This information is based on a telephone conversation with Mr. Rolf Krueger at General Dynamics in Fort Worth.) Although there are no other cost differences between the two models, this adjustment appears to be reasonable with respect to the available data.

C. Parameter Estimation

The parameter estimation procedure is complicated by the unavailability of some data series. The state variable Z(t), which is a transformation of the stock of knowledge, is not observable. Therefore, it is impossible to estimate the parameters in (3.112) in its present form.

The value that is observable in the data is

$$C = \int_{t0}^{t1} x(t)dt,$$

i.e., data is available on the change in cumulative cost per unit time. (Recall that relative prices are assumed constant. This assumption allows cost to be measured in units of the variable resource.) The model must be rewritten in such a way that the predicted values, C', are changes in cumulative cost per unit time. The objective is to find parameter values that minimize $\Sigma(C_i-C_i')^2$. An outline of the solution procedure is as follows:

1. guess initial values for all of the parameters,
2. rewrite (3.112) so that t is an inverse function of x(t) in lieu of Z(t),
3. use t = f(x) to generate the model predicted values C',
4. use nonlinear least squares to find the parameter values such that $\Sigma(C_i-C_i')^2$ is minimized.

The first step, the initialization step, is based on *a priori* knowledge about similar models. If no knowledge is available, the initial values are guesses. The second step requires some additional theoretical development. The objective of the second step is to rewrite (3.112) so that t is a function of x. With this transformation the model is in terms of variables that are observable, i.e., x is integrated with respect to time in order to obtain cumulative cost over an interval of time. The actual cumulative cost over an interval of time is available as a data series.

The transformation proceeds as follows. The total resource requirement rate is the sum of the individual resource requirement rates, i.e.,

$$x(t) = x_q(t) + x_\ell(t). \tag{4.1}$$

After substituting (3.59) and (3.60) into (4.1) and using (3.64), the combined resource requirement function may be rwritten as

$$x(t) = q^\gamma(t)\, a_1^{-\gamma} A^{-\alpha\gamma/(1-\delta)}(t) + z^\beta(t) a_2^{-\alpha}. \tag{4.2}$$

The strategy is to eliminate q(t) and z(t) from the above expression. The leaves an expression which may be solved for Z(t) as a function of x(t).

The following procedure is used to eliminate z(t). Since P(t) = z(t), equation (3.94) implies that

$$z^{\beta}(t) = A\beta Z^{\eta}(t)/\eta + k_2. \tag{4.3}$$

If this result is substituted into (4.2), z(t) is eliminated. The resources required may now be written as

$$x(t) = q^{\gamma}(t)a_1^{-\gamma}z^{-\alpha\gamma/(1-\delta)}(t) + a_2^{-\beta}A\beta Z^{\eta}(t)/\eta + k_2a_2^{-\beta}. \tag{4.4}$$

Now, to eliminate q(t), use the Lagrange-Euler equation (3.82). Solve the Lagrange-Euler equation q(t), and substitute into (4.4) to obtain the desired result. The expression for x(t) as a function of Z(t) is

$$x(t) = (D+E)Z^{\eta}(t) + K_2a_2^{-\beta}. \tag{4.5}$$

The constants are defined as follows:

$$D = k_1^{\gamma}a_1^{-\gamma}, \tag{4.6}$$

$$E = A\beta a_2^{-\beta}/\eta, \tag{4.7}$$

and

$$k_2 = -A\beta Z^{\eta}(T)/\eta. \tag{4.8}$$

The value for k_1 is defined using (3.106) and (3.111). The value of A is also found by algebraic manipulation of

(3. 111) After completing the necessary algebra the constants are found to be

$$D = R^{\beta\gamma/(\beta-\gamma)} Z^{\eta\gamma/(\beta-\gamma)} (T)\eta^{-\gamma/(\beta-\gamma)}$$

$$(\beta-1)^{-\gamma/(\beta-\gamma)} a_2{}^{\beta\gamma/(\beta-\gamma)}$$

$$[\alpha\gamma/(1-\delta)]^{\gamma/(\beta-\gamma)}$$

$$a_1{}^{-\beta\gamma/(\beta-\gamma)} (1-\delta)^{-\eta\gamma(\gamma-1)/(\beta-\gamma)}, \tag{4.9}$$

$$E = -Z^{\eta\gamma/(\beta-\gamma)} (T)\eta^{-\beta/(\beta-\gamma)} R^{\beta\gamma/(\beta-\gamma)}$$

$$(\beta-1)^{-\beta/(\beta-\gamma)} a_2{}^{\beta\gamma/(\beta-\gamma)} a_1^{-\beta\gamma/(\beta-\gamma)}$$

$$[\alpha\gamma/(1-\delta)]^{\beta/(\beta-\gamma)} (1-\delta)^{-\eta\beta(\gamma-1)/(\beta-\gamma)} \tag{4.10}$$

and

$$k_2 = Z^{\eta\beta/(\beta-\gamma)} (T)\eta^{-\beta/(\beta-\gamma)} R^{\beta\gamma/(\beta-\gamma)}$$

$$(\beta-1)^{-\beta/(\beta-\gamma)} a_2{}^{\beta^2/(\beta-\gamma)} a_1{}^{-\beta\gamma/(\beta-\gamma)}$$

$$[\alpha\gamma/(1-\delta)]^{\beta/(\beta-\gamma)} (1-\delta)^{-\eta\beta(\gamma-1)/(\beta-\gamma)} \tag{4.11}$$

Equation (4.5) may be solved for $x(t)$ as a function $Z(t)$, that is,

$$Z(t) = (D+E)^{-1/\eta}[x(t)-k_2a_2{}^{-\beta}]^{1/\eta}. \tag{4.12}$$

Continuing with the solution, substitute (4.12) into (3.112) and to yield t as a function of $x(t)$. The expression that links optimal $x(t)$ with t is

$$t = Z^{(\beta-\gamma-\eta)/(\beta-\gamma)}(T)\eta^{(1-\beta+\gamma)/(\beta-\gamma)}$$

$$R^{-\gamma/(\beta-\gamma)}(\beta-1)^{1/(\beta-\gamma)}a_2^{-\beta/(\beta-\gamma)}a_1^{\gamma/(\beta-\gamma)}$$

$$[\alpha\gamma/(1-\delta)]^{-1/(\beta-\gamma)}(1-\delta)^{\eta(\gamma-1)/(\beta-\gamma)}$$

$$\int_{Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}}^{Z^{-\eta}(T)(D+E)^{-1}[x(t)-k_2a_2^{-\beta}]}(1-y)^{(1-1/\beta)-1}y^{1/\eta-1}dy. \tag{4.13}$$

This expression gives t as a function of x(t) at any instantaneous point in time. Although the expression is optimal, the function is still not appropriate for estimation purposes since the quantity that is observable is not x(t). Cumulative x(t) over an interval is observable, i.e.,

$$C = \int_{t0}^{t1} x(t)dt.$$

Equation (4.13) must be transformed so that it is compatible with the data.

The transformation procedure is as follows. By definition, it is known that

$$X(t) = \int_0^t x(t)dt. \tag{4.14}$$

It follows from (4.5) that

$$X(t) = \int_0^t [(D+E)K_1^{-1}Z^{\eta}(t) + k_2 a_2^{-\beta}]dt. \tag{4.15}$$

This is an expression that links transformed knowledge with cumulative resource usage. By using the second Lagrange-Euler equation (3.82), it is possible to link cumulative resource usage with production rate. The appropriate expression is

$$X(t) = \int_0^t [(D+E)k_1^{-1}q(t) + k_2 a_2^{-\beta}]dt. \tag{4.16}$$

Integrate (4.16) with respect to t to obtain X(t) as a function of Q(t). The expression is

$$X(t) = (D+E)k_1^{-1}Q(t) + k_2 a_2^{-\beta}t + k_5 \tag{4.17}$$

where k_5 is a constant of integration. Since cumulative resource usage is zero at the beginning of the program, X(0) = 0, and by (3.5), $k_5 = 0$. Since Q(t) is known for any value of Z(t) by equation (3.110), and Z(t) is known for any value of t by equation (3.112), X(t) is known for any value of t.

The expression (4.17) is the estimable relationship for this model. An outline of the steps in the estimation process is as follows:

1. determine initial values for the parameters and the beginning stock of knowledge,

2. use equation (3.112) to generate Z(t) for each value of t given by the data,

3. use the value of Z(t) from the previous step to generate a value for Q(t) by using (3.110),

4. use Q(t) in (4.17) to estimate the model parameters by nonlinear least squares.

The final estimable function links cumulative resource usage over an interval with cumulative output and time. The only quantities that are observable X(t) and t, but this presents no problem since Q(t) is generated from the given value of t.

D. Additional Data Adjustments

The expanded theoretical development in the previous section shows that additional data adjustment is needed before the parameters are estimated in the theoretical model. The data in Appendix B is cost by airframe. The data that is needed for estimation purposes is cost per unit time. The ideal data would be cost per airframe per month, but this data is not available. Even if this data were available, the model as presented is not capable of explaining this type of cost behavior. This level of disaggregation would require extensive additional theoretical development. The next best alternative is cost by lot per month. The F102 cost history provides information that makes it possible to assign airframes to lots. The delivery data for each airframe is known, so if the lot release dates were known, and the monthly completion distribution for each lot were known, then it would be possible to generate a cost per month value to use as the dependent variable in the nonlinear regression.

Unfortunately, (based on information obtained from Mr. Rolf Krueger at General Dynamics in Fort Worth) the lot release dates are unknown. However, there is still some available data that makes it possible to approximate the lot release dates for lots four through eleven. Tables 4 and 5 are a reproduction of information provided by Mr. Krueger. These tables give percent completion by lot by month. The tables are segregated into two sections: details and assemblies. These sections are clarified by the information reproduced in Table 6. This table gives the production labor hour summary for the F102 and TF102 by contract. This table is not quite complete. Data was not available on 48 aircraft, but since the quantities of interest are percents of the total hours, the error should be small.

The fabrication hours in Table 6 are the details that are represented in Table 4. For the F102, details or fabrication hours comprised approximately 20% of the total hours expended. Assemblies, in Table 5, are comprised of the four assembly categories in Table 6: major assembly, sub assembly, primary assembly, and final assembly. For the F102, assembly hours accounted for approximately 61% of the total hours expended. The remainder of Table 6, field operations and electronics, are activities that occurred outside of the factory.

TABLE 4. F102 Details Percent Completion by Lot, by Month.

1955

	J	F	M	A	M	J	J	A	S	O	N	D
lot 4	20	20	9	9	17	10	13	2				
lot 5		2	15	8	20	15	25	7	4	2	1	1
lot 6					3.4	8.6	12	5	18	23	17	5
lot 7									4	4	31	24
lot 8												
lot 9												
lot 10												
lot 11												

1956

	J	F	M	A	M	J	J	A	S	O	N	D
lot 4												
lot 5												
lot 6	4	4										
lot 7	20	10	5	2								
lot 8	8	25	40	25	2							
lot 9				8	25	40	26	1				
lot 10					5	30	40	20	8			
lot 11								10	32	48	10	

TABLE 5. F102 Assemblies Percent Completion by Lot, by Month.

1955

	J	F	M	A	M	J	J	A	S	O	N	D
lot 4		5	15	10	12	23	15	3	2	10	5	
lot 5				5	15	15	20	10	15	10	5	3
lot 6						5	5	15	15	20	15	10
lot 7											10	20
lot 8												
lot 9												
lot 10												
lot 11												

1956

	J	F	M	A	M	J	J	A	S	O	N	D
lot 4												
lot 5	2											
lot 6	10	5										
lot 7	25	20	15	10								
lot 8	2	25	35	30	7							
lot 9				4	20	40	30	6				
lot 10						20	40	30	10			
lot 11							5	35	45	15		

TABLE 6. Production Labor Hour Summary for the F102 and TF102 by Contract.

	Contract			
	23903	29264	31774	33685
Fabrication	546,448	1,022,080	2,539,461	524,060
Sub Assembly	328,331	641,843	2,626,727	531,686
Major Assembly	1,082,028	1,928,816	6,155,937	959,650
Primary Assembly	194,969	282,128	804,416	126,318
Final Assembly	149,095	268,258	853,890	130,216
Field Operation	112,895	431,869	1,639,303	328,418
Electronics	72,712	239,996	1,134,832	236,752
Total	2,486,478	4,814,990	15,754,566	2,836,600

The first stage in the data adjustment requires that the total direct manhours per airframe be segregated into three parts: details, assemblies, and outside of the factory. The percentages by contract provided by Table 6 are used to segregate the data. These percentages are presented in Table 7. Next, the percentage that is due to activities outside of the factory is deleted. The reason for this deletion is that there is no information about the monthly distribution of the work that occurred outside of the factory. This leaves total cost by airframe that is due to details and assemblies.

After aggregating into lots, total lot cost is spread over the months using the data in Tables 4 and 5. Implicit in this procedure is the assumption that the lot release date for lots four through eleven may be represented by the first month that activity occurs in the

TABLE 7. Percent of Total Manhours Allocated to Specific Activities by Contract.

	Contract				
	5942*	23903	29264	31174	33965
Fabrication	19.45%	21.98%	21.23%	16.12%	18.47%
Assembly	65.82%	70.56%	64.82%	66.27%	61.62%
Outside of Factory	14.73%	7.46%	13.95%	17.61%	19.91%

*Data on contract number 5942 is not available. The numbers presented are based on averages over the remaining contracts.

respective lot. Lot cost is multiplied by the percentages in Tables 4 and 5 to obtain a monthly cost figure.

Since the model considers cost over the complete project, as a final adjustment, cost must be aggregated by month. This gives a monthly cost value for lots four through eleven which is used as the dependent variable for the analysis. There are clearly some problems with this data adjustment procedure. The initial months over which learning has an important impact on cost are deleted. Also, the deletion of the latter months of the project interjects bias since the production of lots four through eleven is influenced by anticipation of additional production activity in later lots. For example, a close examination of Tables 4 and 5 shows that after August, 1956 not only is production activity taking place on lots ten and eleven, but it is reasonable to assume that

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production has already begun on lot twelve. The implication is that what production occurs in September and October is not likely to be independent of what happens in later lots. Also, the monthly cost will be severely understated. The only reasonable assumption is to delete all activity past August, 1956, and for estimation purposes use the planned volume and the terminal time for the complete project. The cost per month values that are used in the estimation are presented in Table 8. There is no doubt that this is a severe restriction on the empirical model, but additional data on the latter months of the project is just not available.

TABLE 8. *Monthly Data on Direct Manhours (Cost) for Lots Four Through Eleven.*

Cost	Month
87588.960120	1
169255.12250	2
319441.75736	3
283989.79977	4
538366.96822	5
724143.06890	6
735400.38838	7
395148.28286	8
487693.18317	9
605107.55834	10
639831.67327	11
550857.44587	12
638146.06308	13
790801.39694	14
811728.47136	15
756117.01045	16
740850.61671	17
1618970.9276	18
1618970.9276	18
1684015.1252	19
883581.09883	20

E. Econometric Specification

Since the data for this study is monthly data, the appropriate representation of (4.17) requires integrating (4.16) from t_0 to t_1, where the difference between t_0 and t_1 is one month. The appropriate specification is

$$X(t_1)-X(t_0) =$$

$$C_1 \int_{C_4}^{Z^{-\eta}(t)Z^{\eta}(t_1)} y^{(1/\eta+1)-1}(1-y)^{(1-1/\beta)-1}dy \; -$$

$$C_1 \int_{C_4}^{Z^{-\eta}(T)Z^{\eta}(t_0)} y^{(1/\eta+1)-1}(1-y)^{(1-1/\beta)-1}dy \; +$$

$$C_2(t_1-t_0) \tag{4.18}$$

where

$$t = C_3 \int_{C_4}^{Z^{-\eta}(T)Z^{\eta}(t_i)} y^{1/\eta-1}(1-y)^{(1-1/\beta)-1}dy, \tag{4.19}$$

$$C_1 = (D+E)K_1^{-1}, \tag{4.20}$$

$$C_2 = K_2 a_2^{-\beta}, \tag{4.21}$$

$$C_3 = Z^{(\beta-\gamma-\eta)/(\beta-\gamma)}(T)\ R^{-\gamma/(\beta-\gamma)}$$

$$\left(\frac{\alpha\gamma}{1-\delta}\right)^{-1/(\beta-\gamma)}(\beta-1)^{1/(\beta-\gamma)} a_2^{-\beta/(\beta-\gamma)}$$

$$a_1^{\gamma(\beta-\gamma)}(1-\delta)^{\eta(\gamma-1)/(\beta-\gamma)} \tag{4.22}$$

$$C_4 = Z^{-\eta}(T)[M^{1-\delta}/(1-\delta)]^{\eta}. \tag{4.23}$$

This reparametrization is motivated by our previous experience in estimating the parameters in complex nonlinear models Womer and Gulledge [34]. This experience follows from studying the correlation matrix of the estimated coefficients. If one or more of the values in this matrix are large, the implication is that there are too many parameters included in the model. This does not mean that the model is inappropriate, but it is an indication of a data problem, i.e., the data is just not adequate for estimating all of the model parameters. The search for reparameterizations to improve the situation is not always apparent, so this specification (4.18) may require additional reparameterizations depending upon the nature of the data.

F. Initial Conditions and Estimation

The initial conditions for the estimation procedure were determined by selecting reasonable values for η and beta; guessing values for C_1, C_2, C_3, and C_4; and plotting the function until a reasonable a priori "fit" was discovered. This procedure is somewhat arbitrary, but there is no additional information about the complex reparameterized values which would suggest a more theoretical approach. The initial estimation of all six parameters indicates that the model is still overparameterized, i.e., the asymptotic correlation matrix of the parameters indicates that beta cannot be estimated independently from the other parameters. To alleviate this problem, beta is fixed at the final estimated value, and the remaining five parameters are estimated.

The estimation of the five parameter model indicates that the model is still extremely overparameterized. The results of this estimation are presented in Table 9 along with the asymptotic correlation matrix in Table 10. The extreme high correlation among the parameters impacts the estimated standard errors of the parameters in a fashion that is similar to that of multicollinearity in linear regression. The standard errors are inflated to such an extent that it appears that all of the parameters are statistically insignificant, but a comparison of the regression and total sum of squares indicates that a high percentage of the total sum of squares is explained by the regression. For the results of this estimation to be useful, it is important that the high correlations among the parameters be reduced. In the absence of better data,

TABLE 9. Nonlinear Estimation of the Five Parameter Model with β Fixed at 1.01300001.

Source	D.F.	Sum of Squares	Mean Square
Regression	5	10743197176630.214	2148639435346.043
Residual	15	1560091561904.984	104006104126.999
Total	19	12303288738535.198	

Parameter	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval
C1	96066380.904	185595987954.9616	-395490913877 to 395683046649
C2	-98075.835	614152796.7175	-1309128798 to 1309032647
C3	198368.218	1207159126792.4570	-2572988792037 to 2572989188774
C4	.079	229.6572	-489 to 489
η	4.096	15750.2727	-33566 to 33574

TABLE 10. Asymptotic Correlation Matrix Associated with the Nonlinear Estimation of the Five Parameter Model with β Fixed at 1.01300001.

	c_1	c_2	c_3	c_4	η
c_1	1.00	.99	.99	.99	-.99
c_2	.99	1.00	.99	-.99	-.99
c_3	.99	.99	1.00	-.99	-.99
c_4	-.99	-.99	-.99	1.00	.99
η	-.99	-.99	-.99	.99	1.00

the only alternative is to "fix" values for some of the parameters while estimating the remaining less correlated parameters.

The following approach was taken to identify the estimable relationships associated with this set of data. Since the model is highly nonlinear and the collinear relationships among the variables are not obvious, all possible two parameter models were estimated in order to identify pairs of parameters which may be included in the estimation. This was accomplished by fixing the values of the remaining parameters at the estimated values presented in Table 9. This approach has shortcomings in that it only considers pairwise correlations among parameters, but because of the severe nonlinearities this appears to be a plausible approach. The results of these two parameter estimations are presented in Table 11.

A close examination of all of the regression results indicates that the standard errors are extremely inflated on many of the two parameter models. In fact, the asymptotic standard errors are meaningless on all but five of the models. The complete regression diagnostics for each of these models is presented in Tables 12 through 16. The results of the estimations are surprisingly stable with the exception of the estimate for C_2. In all of the cases where C_2 and C_4 are estimated, they are not significantly different from zero. However, these results must be interpreted with care because of the nature of this estimation, and since the data associated with lots one through three was deleted. The signs and magnitudes of all the parameters agree with a priori expectations, but again the results must be interpreted with care.

Even though these results are tenuous at best, there is something to be learned from these estimations. After examining Table 11, it appears that there are many alternative specifications which are approximately the same in terms of sum of squares reduction with this set of data.

Since there are several two parameter models that are adequate, the logical extension is to search for an estimable three parameter model. Three parameter models are selected from the two parameter models with the lowest pairwise correlations between parameters.

TABLE 11. Summary Results of Regressions Containing Two Parameters.

Parameters	ESS*	Asymptotic Correlation Between Parameters
C_1 and C_2	100.11	-.8612
C_1 and C_3	306.99	-.9999
C_1 and C_4	100.46	-.9945
C_1 and η	102.00	.9961
C_1 and β	100.33	.9987
C_2 and C_3	309.46	.9999
C_2 and C_4	100.43	.9999
C_2 and η	99.77	.8576
C_2 and β	100.32	.8788
C_3 and C_4	309.17	1.0000
C_3 and η	99.81	-.9996
C_3 and β	100.33	1.0000
C_4 and η	99.76	.9900
C_4 and β	100.33	.9893
η and β	99.79	-.9998

* ESS is divided by 1×10^9.

TABLE 12. Regression Results of Cost on Time with C_1 and C_2 as Parameters.

Source	D.F.	Sum of Squares	Mean Square
Regression	2	11302133353373.123	5651066676686.561
Residual	18	1001155385162.075	55619743620.115
Total	19	12303288738535.198	

Parameter	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval
C_1	1919601903.523	312890412.578	1262247883 to 2576955923
C_2	130169.033	103781.921	-87867 to 348205

Asymptotic Correlation Matrix of the Parameters

	C_1	C_2
C_1	1.0000	-.861
C_2	-.861	1.0000

TABLE 13. Regression Results of Cost on Time with C_2 and η as Parameters.

Source	D.F.	Sum of Squares	Mean Square
Regression	2	11305585907011.182	5652792953505.591
Residual	18	997702831524.017	55427935084.668
Total	19	12303288738535.198	

Parameter	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval
C_2	-5690.700	102382.686	-220787 to 209405
η	8.724	.771	7.10 to 10.34

Asymptotic Correlation Matrix of the Parameters

	C_2	η
C_2	1.0000	.857
η	.857	1.0000

TABLE 14. Regression Results of Cost on Time with C_2 and β as Parameters.

Source	D.F.	Sum of Squares	Mean Square
Regression	2	11300067555099.707	5650033777549.858
Residual	18	1003221183435.491	55734510190.861
Total	19	12303288738535.198	

Parameter	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval
C_2	6970.392	110626.69566	-225446.1346 to 239386.9201
β	1.017	.00043	1.0170 to 1.0188

Asymptotic Correlation Matrix of the Parameters

	C_2	β
C_2	1.000	.878
β	.878	1.000

TABLE 15. Regression Results of Cost on Time with η and C_4 as Parameters.

Source	D.F.	Sum of Squares	Mean Square
Regression	2	11305675666179.633	5652837833089.816
Residual	18	997613072355.565	55422948464.198
Total	19	12303288738535.198	

Parameter	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval
η	8.556	2.638147	3.013998 to 14.099011
C_4	.000066	.000046	-.000030 to .000163

Asymptotic Correlation Matrix of the Parameters

	η	C_4
η	1.000	.990
C_4	.990	1.000

TABLE 16. Regression Results of Cost on Time with C_4 and β as Parameters.

Source	D.F.	Sum of Squares	Mean Square
Regression	2	11299935382561.825	5649967691280.912
Residual	18	1003353355973.373	55741853109.632
Total	19	12303288738535.198	

Parameter	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval
C_4	.00007158	.00008119	-.000098 to .000242
β	1.01797696	.00140531	1.015024 to 1.020929

Asymptotic Correlation Matrix of the Parameters

	C_4	β
C_4	1.000	.989
β	.989	1.000

Table 17 provides summary information from these regressions. A comparison with Table 11 shows that no appreciable reduction in residual sum of squares is obtained by considering the three parameter model. Also, none of the three parameter regressions provided estimates that were suitable for making inferential statements about the true population values. In all cases the standard errors are inflated due to the extreme correlations among the parameters.

Since the three parameter combinations in Table 17 represent a small subset of all possible three parameter combinations, additional three parameter models were selected for estimation. Because of the large number of

TABLE 17. Summary Results of Regressions Containing Three Parameters.

Parameters Estimated	ESS*
C_1, C_2 and η	99.76
C_1, C_2 and β	100.10
C_1, C_4 and η	99.68
C_1, C_4 and β	100.32
C_1, η and β	99.58
C_2, C_4 and η	99.72
C_2, C_4 and β	100.07
C_2, η and β	99.62
C_4, η and β	99.53

* ESS is divided by 1×10^9.

combinations, it is not practical to estimate all three parameter combinations, so a selection of models containing different combinations of all six parameters was estimated. The results were the same in all cases. There was no appreciable reduction in sum of squares, and all of the models had inflated standard errors. The results of all of the regressions indicate that a two parameter model is adequate for explaining this data.

As a test of this hypothesis the same set of data was used to estimate the parameters in the model presented by Womer [38]. This model is a four parameter model, and it is estimated in the form presented in equation (17). As in the previous case, the estimation results indicate that there are more parameters in the model than are necessary to represent this data adequately. After numerous reparameterizations, the following specification was selected as the only estimable model:

$$X(t_1)-X(t_0) = \beta_0[e^{\beta_1 t_1} - e^{\beta_1 t_0}]. \qquad (4.24)$$

If the model is reparameterized in this form, $\beta_1 = \rho\gamma/(\gamma-1)$, and β_0 is a constant term. If ρ is fixed at a suitable value, an estimation of (4.24) yields some information about the scale parameter γ. Unfortunately, the reparameterization required by the data makes it impossible to obtain any information about δ. The discount rate is fixed at .008333, a number that is consistent with an assumed 10% annual rate. The parameters were estimated using nonlinear least squares, and convergence was obtained in 4 iterations after using a grid search procedure to establish the initial parameter values. The results of the estimation on (4.24) are presented in Table 18.

These results support previously presented research which stresses the importance of the cost impact of production rate changes in airframe programs. An asymptotic 95% confidence interval shows that the scale parameter is significantly greater than one, indicating diminishing returns to the variable factor. The other paramter, β_0, is not significantly different from zero at the 95% level, but it is significant at the 90% level.

These results confirm the previous hypothesis about the F102 data. As previously stated, both of the models may be appropriate in terms of theoretical specification, but the F102 data is not sufficient for estimating many parameters. It appears that the data is not capable of supporting more than two parameters.

The final stage of the modeling process requires selecting a model from among the estimable two parameter models. This selection is complicated because there is no a priori justification for favoring any particular model. A desirable selection would be one that is comparable with the 1979 model presented by Womer [38]. A study

TABLE 18. Nonlinear Least Squares Summary Statistics for Reparameterized Model.

Source	D.F.	Sum of Squares	Mean Square
Regression	2	11308925768492.965	5654462884246.482
Residual	18	994362970042.234	55242387224.569
Total	20	12303288738535.126	

Parameter	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval
β_0	852936.	553864.	-310681 to 2016554
γ	1.1089	.022	1.06 to 1.15

Asymptotic Correlation Matrix of the Parameters

	β_0	γ
β_0	1.000	.994
γ	.994	1.000

of possible two parameter models indicates that the model containing C_1 and β would probably be best for comparison purposes, but that model has inflated standard errors. Two possible choices are presented in Tables 14 and 15. The model containing C_4 and β is selected since inferences about C_2 (this model is contained in Table 14) are uninteresting, i.e., C_2 is the coefficient on an additive constant term, and the initial sample points were deleted.

As one last step before proceeding to inference and diagnostic checking, the model in Table 16 reestimated after completing a tight grid search around the remaining parameters while holding C_4 and β constant at their estimated values. This one last step is to see if any further reduction in error sum of squares may be obtained by altering the fixed values in the original regression. This procedure yielded a very slight reduction in sum of squares, and the changes in the parameter values were very slight. The final estimation results are presented in Table 19, and a summary of all of the parameter values (including fixed) are presented in Table 20.

G. Diagnostic Checking

To determine how well the model presented in Table 19 conforms with the theoretical assumptions associated with the estimation, a series of diagnostic test are performed. These tests are basically of two types: visual and inferential. The visual checks are composed of various plotting

TABLE 19. Final Estimation of the Two Parameter Model Containing β and C_4.

Source	D.F.	Sum of Squares	Mean Square
Regression	5	11299917288762.562	11299917288762.562
Residual	15	1003371449772.636	52809023672.244
Total	19	12303288738535.198	

Parameter	Estimate	Asymptotic Standard Error	Asymptotic 95% Confidence Interval
C_4	.00007252	.00008219	-.00010015 to .00024520
β	1.01799282	.00140706	1.01503672 to 1.02094824

Asymptotic Correlation Matrix of the Parameters

	C_4	β
C_4	1.000	.989
β	.989	1.000

TABLE 20. Final Parameter Estimates.

$$C_1 = 20000015102.00000$$

$$C_2 = 812.42534$$

$$C_3 = 197190.69301$$

$$C_4 = .00007$$

$$\eta = 4.10000$$

$$\beta = 1.01799$$

techniques for checking conformance to assumptions, while the inferential checks are formal statistical tests for verifying many of the same assumptions.

The usual assumptions associated with the regression model are that the errors are independent, have zero mean, a constant variance, and follow a normal distribution. If these assumptions are satisfied, the residuals should exhibit characteristics that are consistent with the assumptions. The residual plots are of two types:

1. residuals against time
2. residuals against predicted values.

These plots are presented in Figures 6 and 7. An examination of the sequence plot, Figure 6 reveals that based on this sample, it would be impossible to visually reject the hypothesis that the disturbances are heteroscedastistic and autocorrelated. The imposition of a "band" on the

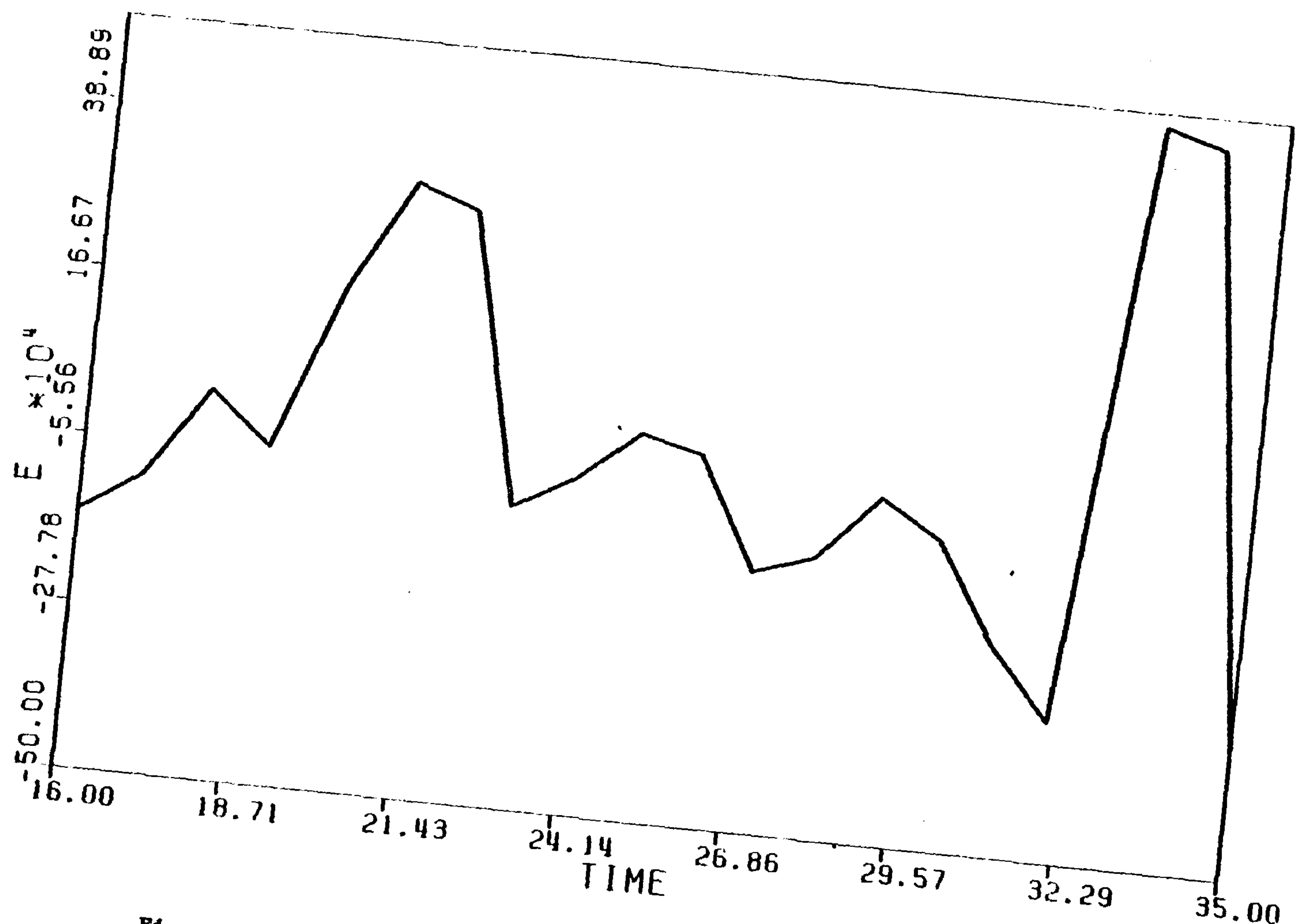

Figure 6. Sequence Plot of Residuals Against Time

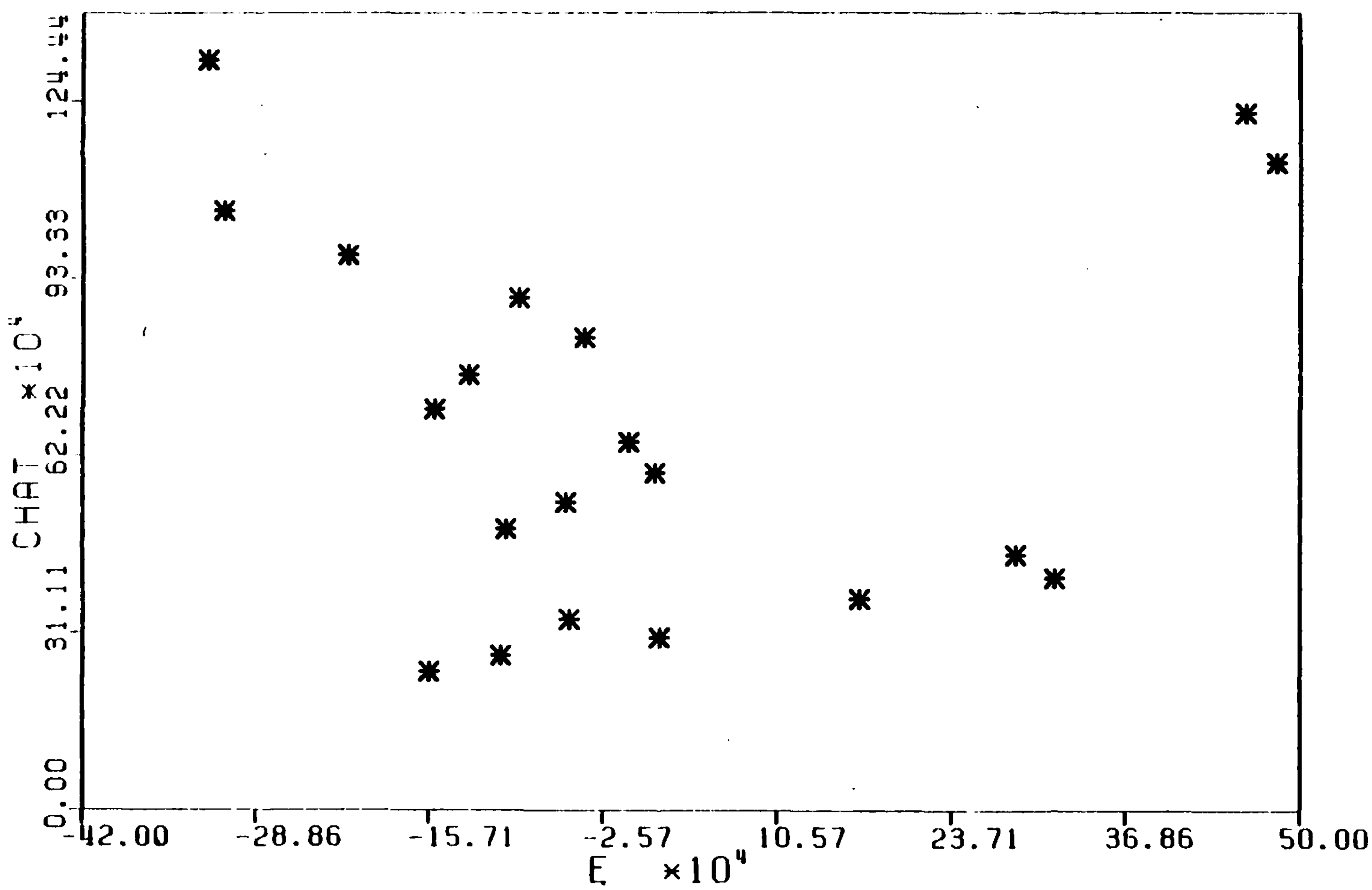

Figure 7. Scatter Plot of Residuals Against Predicted Values for Cost

residuals produces a funnel shape which may indicate heteroscedastistic disturbances. Also, the long positive and negative "runs" above and below the mean may indicate that the disturbances are autocorrelated. The plot of the residuals against the fitted values, Figure 7, also indicates that the model may be deficient. This follows from the systematic pattern that shows that high fitted values are associated with positive residuals and vice versa.

There are several asymptotic results that follow from the summary results in Table 19. The asymptotic 95% confidence intervals for the parameters, C_4 and β yield additional information about goodness-of-fit. The 95% interval for C_4 contains zero, an indication that C_4 is not significantly different from zero. Also, the ratio of the regression to the residual sum of squares gives information about how well the model fits the data. For this estimation, the ratio is approximately .9. This high ratio is confirmed by the plot of the actual versus the fitted values presented in Figure 8. Again it must be stressed that the results must be interpreted with care because of the small sample size.

H. Conclusion

This research is an extension of a general theoretical framework. It represents a cataloging of dynamic models which have potential for application in the airframe industry. Though not exhaustive, it defines and summarizes a whole area of future research.

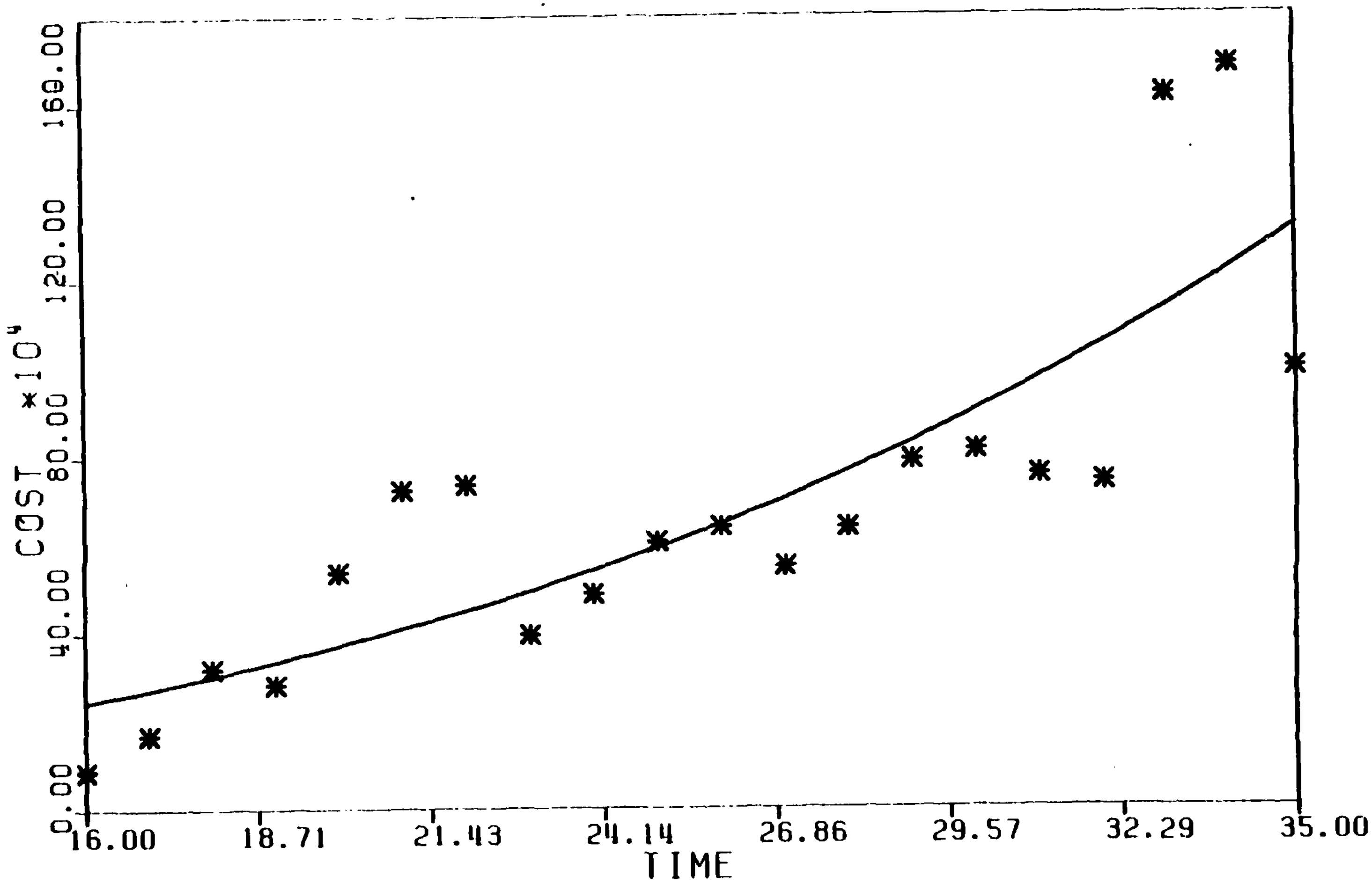

Figure 8. Sequence Plot of Predicted and Actual Values of Cost Against Time

The first general conclusion stresses the importance of reliance on economic theory when formulating models to explain economic phenomena. Empirical cost models may yield results that are meaningless. All of the models presented in this research are firmly grounded in neoclassical economic theory.

The second conclusion concerns the role of output rate in explaining cost. Many learning curve parameters have studied only one dimension of the determinants of cost: cumulative output. The models presented in this research stress the importance of output rate and cumulative output rate as determinants of cost. To obtain meaningful cost estimates, both dimensions must be included in model specification.

The third conclusion concerns the data that is readily available to the cost analysis community. Perhaps because of analysts preoccupation with learning curves, the Air Force cost analysis community does not seem to cause data on the time dimension of costs to be preserved for cost analysis. As a result, even though the F-102 data is in many ways extensive it sheds very little light on production costs over time.

The fourth conclusion concerns the difficulty associated with the application of these models. In many cases the solutions yield highly nonlinear relationships which lead to difficult estimation problems. Even the simplest models require a very rich set of data to obtain information about all of the parameters.

After considering the difficulty associated with application, this approach to modeling the made to order production situation still seems to be most promising. The solutions are difficult, but they are at least in agreement with economic theory. Future research in this area includes the continuing effort to solve the models for which no solution was obtained, and the applications of existing models to new sets of data. Also, as a last suggestion, it may be possible to formulate these models in terms of the dual cost function. If this were the case, it may be possible to obtain simpler relationships which may be easier to apply. Research is proceeding in this area.

V. A REVISED MODEL

A. Introduction

The sensitivity analyses on the model reported in Chapter II suggest that some revision of that model is in order. However, the models of Chapter III are all rather complex. In addition, the simplest of these models has data requirements that are difficult to meet at this time. These results suggest that a model which compromises the theoretical correctness of the models in Chapter III with the realities of data availability and the requirements of program management may be the best practical solution for this problem.

The result of this compromise is the revised model reported below. It is based mainly on the conclusions drawn from the sensitivity analyses of Chapter II and from similar analyses of other modifications to that model. One of the basic changes to the original model is a change in focus. Instead of modeling the production of a batch of airframes, the revised model is focused on the production of a single airframe. In addition, the four effects described at the end of Chapter II are used to form production cost drivers in the revised model.

The first production cost driver is the concept of learning by doing. The basic idea is that as the cumulative number of airframes produced increases the unit costs (or at least labor hours) decreases. This component is the only production cost driver that is sometimes included in parametric cost estimates. It is commonly discussed in both the industrial engineering and the operations research literature, but the learning curve is only rarely mentioned in the economics literature on production and cost.

To aid our thinking about learning and the other production cost drivers, we follow Washburn [33] by adopting the concept of a production line as a frame of reference. Learning by doing affects cost by affecting efficiency at each position on the production line. That is, as the number of airframes passing each position on the line increases, yielding more experience, the efficiency at the position increases, thus lowering labor cost.

Notice that this process implies that at any point in time the experience on the production line may vary dramatically from the beginning to end. (In the C-141 program as much as two years elapsed between the lot release date and delivery of an airframe.)

The second production cost driver is a different learning effect. Over time, learning how to produce more efficiently may take place due to events other than experience at a position on the production line. For example, early in a production program labor hours may be spent to learn how to produce more efficiently. Later in the program this may result in increased efficiency independent of experience at a point on a line. If this is the case, positions at the end of the line work more efficiently on the same airframe than positions at the beginning of the line. Or, this effect may be related to experience at other locations on the production line. That is, positions late in the production line may benefit from the experience of earlier positions, thus work at later positions proceeds more efficiently than work at early positions on the same airframe.

A third production cost driver is the speed of the production line. Unless compensated for by learning, increasing the speed of the line is expected to require more labor at each position on the line. Furthermore, due to diminishing returns, the additional labor required is expected to be more than in proportion to the increase in speed. Anyone who has observed activity around an airframe during production will recognize the likelihood of diminishing returns to labor on that airframe.

The fourth cost driver is the length of the production line. One way to increase delivery rate is to increase the number of positions on the production line, reducing the amount of work to be done at each position, and increasing the total amount of work accomplished per unit of time. If alternative length production lines are planned, this driver may not be a source of variation in unit costs. However, if the length of the line is changed on short notice, unit costs may be affected. For example, increasing the length of the line may result in crowded facilities and overused tools and other fixed resources. This adversely affects the efficiency of production and may result in increased unit costs. This last effect involves an interaction among the airframes that are in the facility at the same point in time.

The model of production described in the next section represents an attempt to capture these effects in an estimable analytic model.

B. The Model

Like the model in Chapter II this model augments a homogeneous production function with a learning hypothesis. The discounted cost of production is minimized subject to a production function constraint to derive the optimal time path of resource use. Since factor prices are assumed to be constant over the relevant time period, cost is measured in the units of the variable resource. The variables used in the analysis are:

i = the sequence number of an airframe, $(i = 1, \ldots, n)$

V = the average number of airframes in process

t_{si} = the date work begins on airframe i; work on all airframes in the same lot is assumed to start on the lot release date,

t_{di} = the delivery date for airframe i,

$q_i(t)$ = the production rate at time t on airframe i,

$Q_i(t)$ = the cumulative work performed on airframe i at time t, i.e.

$$Q_i(t) = \int_{t_{si}}^{t} q(\tau)\, d\tau,$$

$x_i(t)$ = the rate of resource use at time t on airframe i,

δ = a parameter describing learning prior to airframe i,

ε = a parameter describing learning on airframe i

γ = a parameter describing returns to the variable resources,

α = a parameter associated with decreases in labor productivity as an airframe nears completion,

ν = a parameter describing returns to the length of the production line,

ρ = the discount rate,

C = discounted variable program cost,

C_i = discounted variable cost of a single airframe.

The production function is assumed to be of the following form:

$$q_i(t) = A(i-1/2)^{\delta} Q_i^{\varepsilon}(t) \ (t_{di}-t)^{\alpha} x^{1/\gamma}(t) V^{\nu} \tag{5.1}$$

where A is a constant. The input x is assumed to be a composite of many inputs whose rate is variable throughout the production period.

This production function represents an attempt to include the production cost drivers described in the previous section, it conforms to economic production theory, and it also accommodates the fact that the nature of work along the production line changes from position to position. On the other hand it is still a very simple function and it can only be expected to describe such a complex production process with some error.

The term $(i-1/2)^{\delta}$ describes learning by doing in producing the ith airframe. The terms $Q_i^{\varepsilon}(t)$ and $(t_{di}-t)^{\alpha}$ represent attempts to describe learning that occurs over time during the process of producing airframe i. These terms also admit the possibility that the nature of work changes as the airframe moves down the production line. In particular, it is assumed that as the delivery date is approached it is more difficult to substitute labor for time in the production process. and are both expected to be between 0 and 1. However, below it is seen that the effect of learning while producing an airframe and the effect of the learning prior to production cannot be separated with our data, so the ε cannot be estimated.

Still one more term t^h would have been useful to include in the production function to model this effect. Unfortunately, we have been unable to solve the resulting control problem if time is included in this way.

The term $X_{ij}^{1/\gamma}(t)$ captures the effect of the speed of the production line. We expect γ to be greater than 1.

Finally the term V^{ν} is intended to capture the effect of working on alternative numbers of airframes in the same facility. It is assumed that more airframes in the same facility results in a slight decrease in efficiency (ν is negative and small).

Although the objective of the firm is a function of the wording of the contract, one goal of most contracts is to induce the firm to minimize discounted cost. The problem may be stated as:

$$\text{Min } C = \sum_{i=1}^{n} \int_{t_{si}}^{t_{di}} x_i(t) e^{-\rho t} dt \tag{5.2}$$

$$\text{s.t. } q_i(t) = A(i-1/2)^{\delta} Q_i^{\varepsilon}(t) \; (t_{di}-t)^{\alpha} x_i^{1/\gamma}(t) V^{\nu},$$

$$Q_i(t_{di}) = 1, \qquad Q_i(t_{si}) = 0. \qquad (i=1,\ldots n)$$

Since total cost is monotone nondecreasing and the sub-problems are additive, the solution can be obtained by minimizing each of the sub-problems. The representative problem for the ith airframe may then be stated as:

$$\text{Min } C_i = \int_{t_{si}}^{t_{di}} x_i(t) e^{-\rho t} dt \tag{5.3}$$

$$\text{s.t. } q_i(t) = A\,(i-1/2)^{\delta} Q_i^{\varepsilon}(t) \; (t_{di}-t)^{\alpha} x_i^{1/\gamma}(t) V^{\nu},$$

$$Q_i(t_{di}) = 1, \qquad Q_i(t_{si}) = 0.$$

Except for the determination of constants this is the same calculus of variations problem that was solved in Chapter II. The reader is referred for the solution procedure. Following that procedure yields the folllowing resource requirement function:

$$x_i(t) = B\,(i-1/2)^{-\gamma\delta} \Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1), \alpha\gamma(\gamma-1)+1] \\ (t_{di}-t)^{\alpha\gamma/(\gamma-1)} e^{-\gamma\rho(t_{di}-t)/(\gamma-1)} V^{-\gamma\nu} \tag{5.4}$$

where $B = A^{-\gamma}(1-\varepsilon)^{-\gamma}[\rho/(\gamma-1)]^{\alpha\gamma^2/(\gamma-1)+\gamma}$

and $\Gamma(\ ,\ )$ is the incomplete gamma function. This is the optimal time path for resource use on any airframe.

Since the data presented by Orsini [22] is quarterly data, the quantity of interest is the total resource use over a quarterly period. If T_1 and T_2 represent the beginning and ending dates for a quarterly period, then the appropriate expression in airframe i is:

$$X_i(T_2)-X_i(T_1) = \int_{T_1}^{T_2} x_i(t)\,dt, \tag{5.5}$$

and using (5.4) the integral is

$$\begin{aligned} X_i(T_2)-X_i(T_1) = {} & B'(i-1/2)^{-\gamma\delta}\Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1),\alpha\gamma/(\gamma-1)+1] \\ & V^{-\gamma\nu}(T_1,T_2)\{\Gamma[\gamma\rho(t_{di}-T_1)/(\gamma-1),\alpha\gamma/(\gamma-1)+1] \\ & -\Gamma[\gamma\rho(t_{di}-T_2)/(\gamma-1),\alpha\gamma/(\gamma-1)+1]\} \end{aligned} \tag{5.6}$$

Because of the nature of the data it is impossible to observe the quantity on the left side of equation (5.5). What is observable is direct man-hours per lot. This means that the observed quantity is

$$\sum_{i=K_j}^{n_j} [X_i(T_2) - X_i(T_1)] \tag{5.7}$$

where K_j and n_j are the sequence numbers of the first and last airframes in lot j. In this instance, the sum is the observed values of labor hours that are reported in Orsini's data set. This sum and the airframe delivery dates are the variables that are used to estimate the model.

C. Empirical Results

To explore the applicability of the theoretical specification, the parameters in (5.6) are estimated using the C-141 data. This data is described in Chapter II. The only difference in the data used here is the calculation of the airframe delivery date t_{di}. In Chapter II we assumed that all airframes in the same batch were delivered at the midpoint of their delivery month. Here we assign delivery dates by spreading the delivery dates evenly across the delivery month. Otherwise, this is the same data used in Chapter II and reported in Appendix A.

$$B_0 = B'$$

and

$$B_1 = \alpha\gamma/(\gamma-1) + 1.$$

The model may be restated as:

$$\sum_{i=K_j}^{n_j} X_i(T_2)-X_i(T_1) = \sum_{i=K_j}^{n_j} \beta_0(i-1/2)^{-\gamma\delta}\Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1),\beta_1] \\ V^{-\gamma\nu}(T_1,T_2)\{[\gamma\rho(t_{di}-T_1)/(\gamma-1),\beta_1)] \\ -\Gamma[\gamma\rho(t_{di}-T_2)/(\gamma-1),\beta_1]\} \tag{5.8}$$

Equation (5.8) is estimated using nonlinear least squares as implemented by SAS's Proc NLIN [27]. The results of this regression are presented in Table 21. Almost all of the parameter estimates seem to be significantly different from zero. However, the asymptotic standard errors for β_0 and ρ seem to be large. In the case of β_0 a scale parameter, this is not of much concern. If ρ is not much different from zero then the objective function in the optimization problem need not include the exponential term and more appropriate production functions might be used. This is a matter for future investigations. The relatively high asymptotic standard error for ρ should not be interpreted as an indicator that the model does not fit the data well or that it is not correct. The asymptotic standard errors reported are calculated based on the assumption that the model is approximately linear in the parameters in the neighborhood of the estimate. This is extremely unlikely in the case of ρ. Another indicator that ρ is an important parameter is the fact that restricting ρ to be zero produces a model with substantially higher mean squared errors.

Table 21

Parameter Estimates and Asymptotic Standard Errors

Parameters	Estimates	Standard Errors
β_0	1.150	0.688
β_1	3.045	1.162
δ	0.484	0.064
γ	1.002	.004
ν	-0.440	.165
ρ	0.002	.004

$$MSE = 3.66 \times 10^{10}$$

Like the model of Chapter II this functional form generates a time path of resource use for an airframe that conforms to our understanding of that process. Unfortunately, we cannot observe resource use by airframe. We can, however, observe the time path of resource use for the entire program. Figure 9 illustrates the predicted time path of resource use for the program and the actual resources used. While the model fits the data well ($R^2 = .69$), the model shows more variation with time than the data does. This is particularly true for the period between quarter 12 and quarter 18. In this interval the model first predicts that more manhours should be used and then that somewhat fewer manhours should be used. We suspect that this is because the model includes no penalty for hiring or firing costs. Therefore, even though the model predicts that the workforce should rise, then decline, and then rise again, the company (correctly) chose to maintain a more moderate sized workforce over the relatively brief peak and slump in requirements. If this is true, then a more appropriate delivery schedule should have permitted substantial savings on the program. These questions are investigated further in the next section.

D. Sensitivity Analyses

To further illustrate the sensitivity of the model to changes in the delivery schedule we have plotted the time path of resource use (equation (5.4) summed over i) for several alternatives to the actual delivery schedule. Each

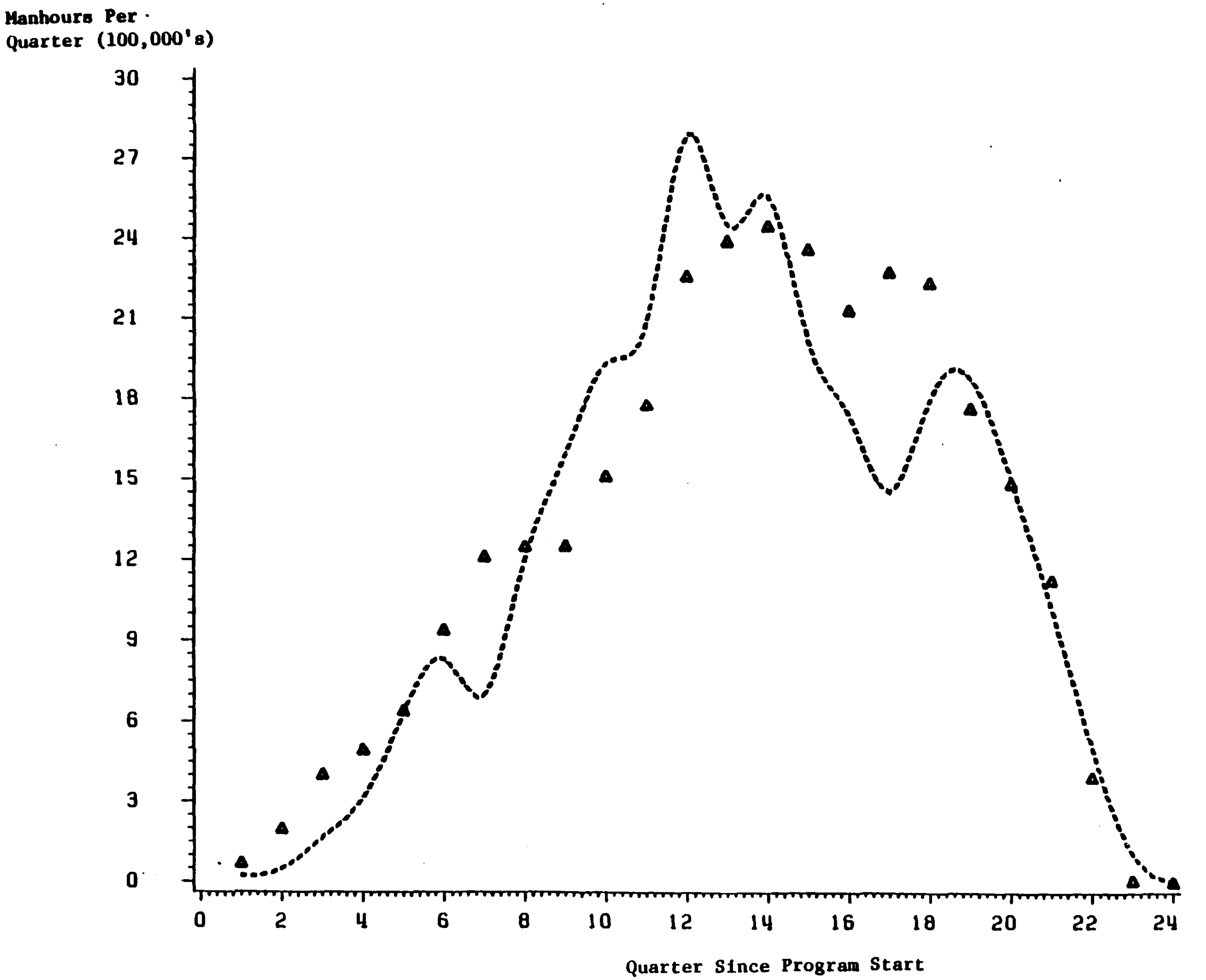

Figure 9. The timepath of resource use

of the alternatives represents a small discrete change to the actual delivery schedule. In the figures which illustrate these changes (Figures 10-14) the actual schedule is represented by a dashed curve and the alternative schedule by a solid curve.

The first alternative, Figure 10, has the first airframe in the program delivered one month later than reported in the actual delivery schedule. This causes the rate of resource use to be lower early in the program but higher as the new delivery date is approached. The net effect is a small increase in predicted program cost. This delivery schedule change operates by adding one month to the first airframe's production time increasing t_{d1}. The time from program start until first delivery ($t_{d1} - t_{j1}$) is also increased. Furthermore, V is increased during quarters five and six. The effect of delaying this delivery increases the learning applicable to the first unit by providing more time prior to delivery, but this effect is offset by the fact that V increases (the number of positions on the production line increases). The net effect is a slight rise in program cost and a delay in program costs (and benefits).

The second sensitivity analysis illustrates the effect of compressing the delivery schedule at the end of the program. Here we consider the effect of delivering the last airframe one month early. This results in reducing the time to work on the last airframe and the time for learning. It also results in an increase in V during the period when the

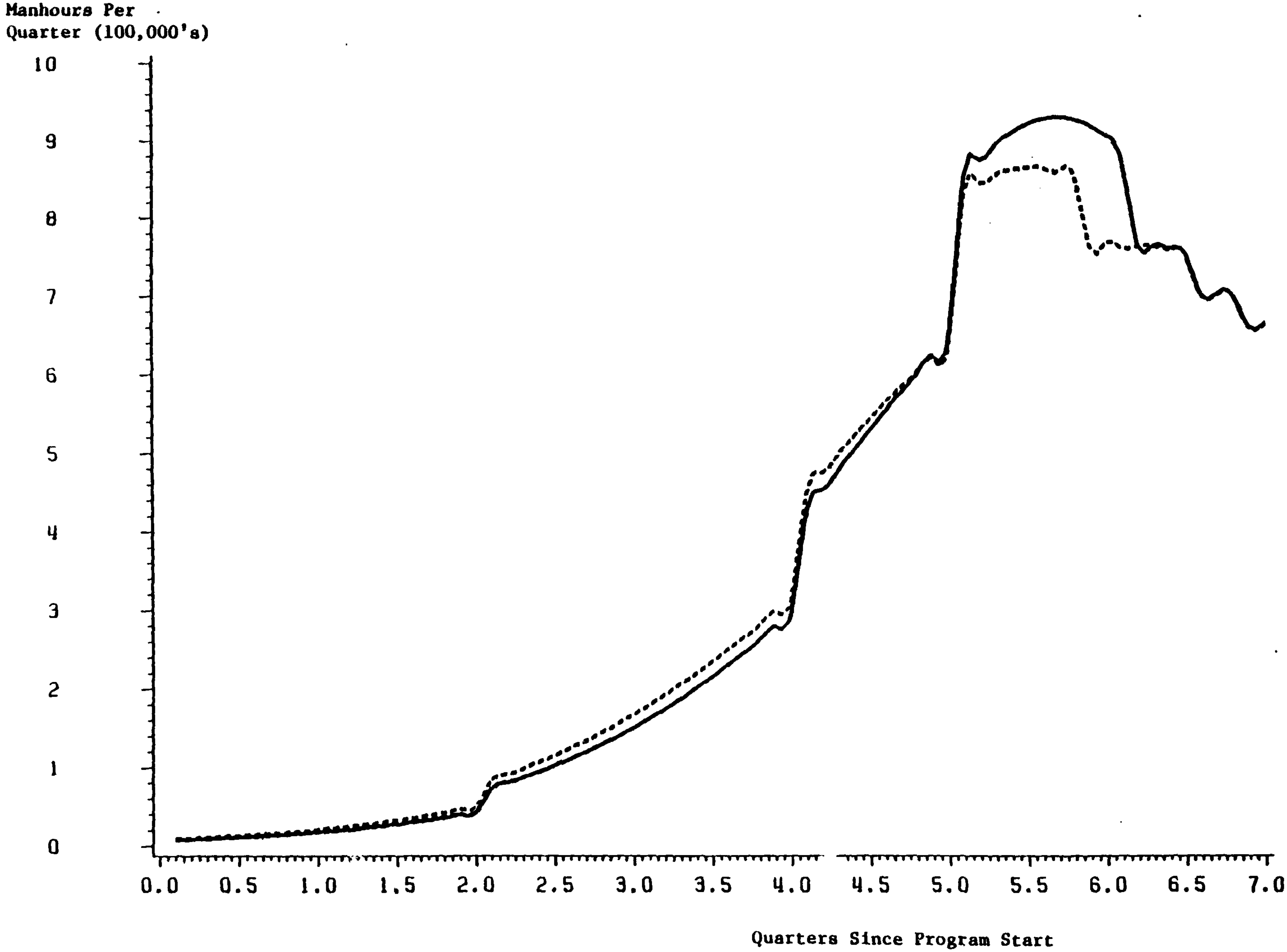

Figure 10. Deliver the First Airframe One Month Later

last airframe was completed. These changes suggest slightly higher program costs for this alternative, and in fact program costs are slightly higher for this change. However, the change is so slight that it isn't indicated in Figure 11

Next we consider advancing the delivery of one airframe in the middle of the program. Between the middle of quarter 14 and the middle of quarter 15 deliveries on the C141 program increased from seven per month to nine per month. Only in the first month of quarter 15 were 8 airframes delivered. In Figure 12 we illustrate the effect of increasing from one month to one quarter this period where the delivery rate was 8. Here we increase deliveries by one in the last month of quarter 14 and decrease them by one in the second month of quarter 15. This decreases the time available for learning, tending to raise cost, but it also decreases V, tending to lower cost. The net effect is to increase resource use on the program up to quarter 14 and to decrease resource use between quarters 14 and 16. This results in a slight decrease in program cost.

We also consider the effect of changing lot release dates on the program. In Figure 13 the lot release date for the last lot is delayed until the start of quarter 16. In addition to preventing expenditures on the last lot in quarter 15, V is lower during that quarter, reducing cost for other lots. Also this effectively shifts the work on the last lot to periods of time when V is lower. These effects

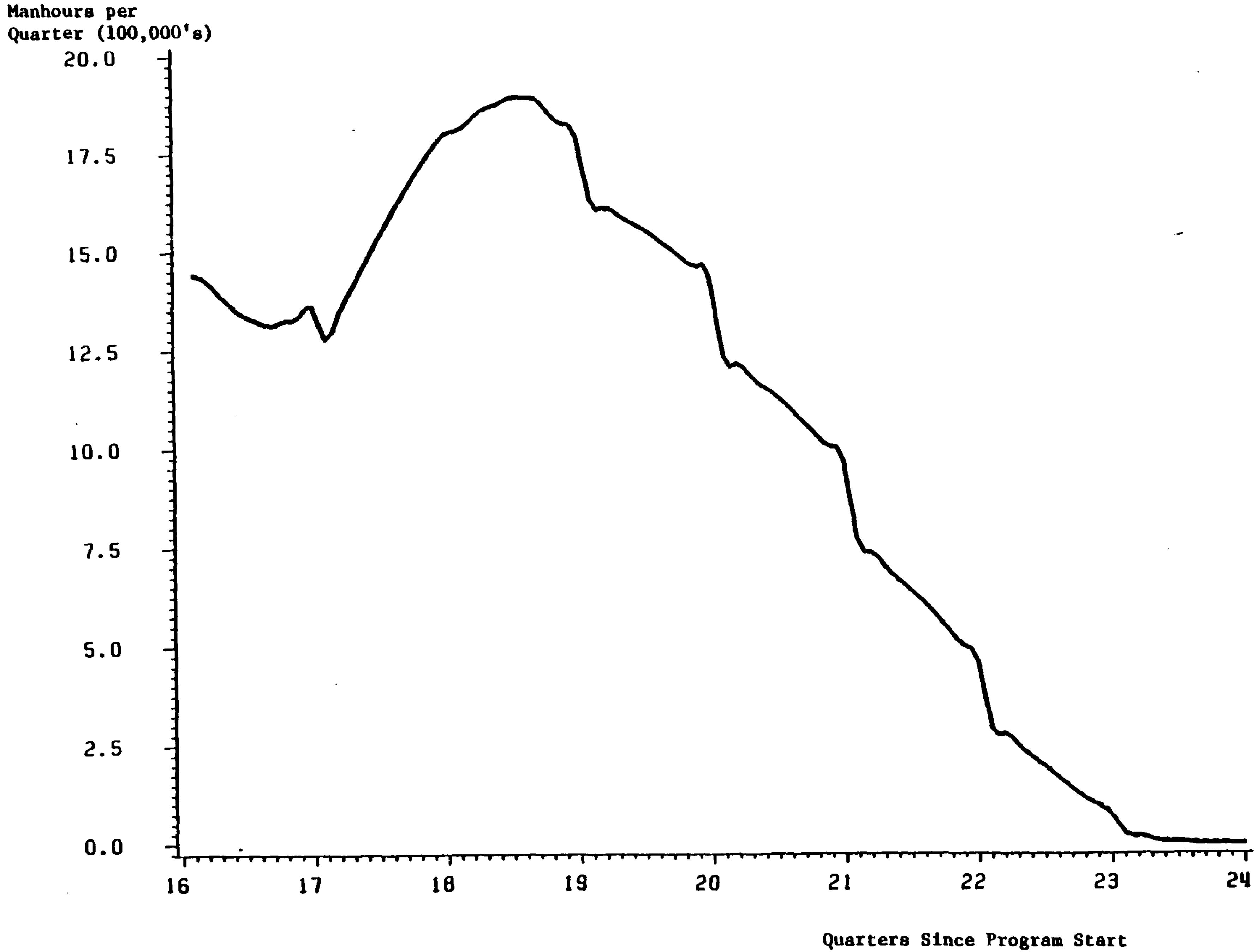

Figure 11. Deliver the Last Airframe One Month Earlier

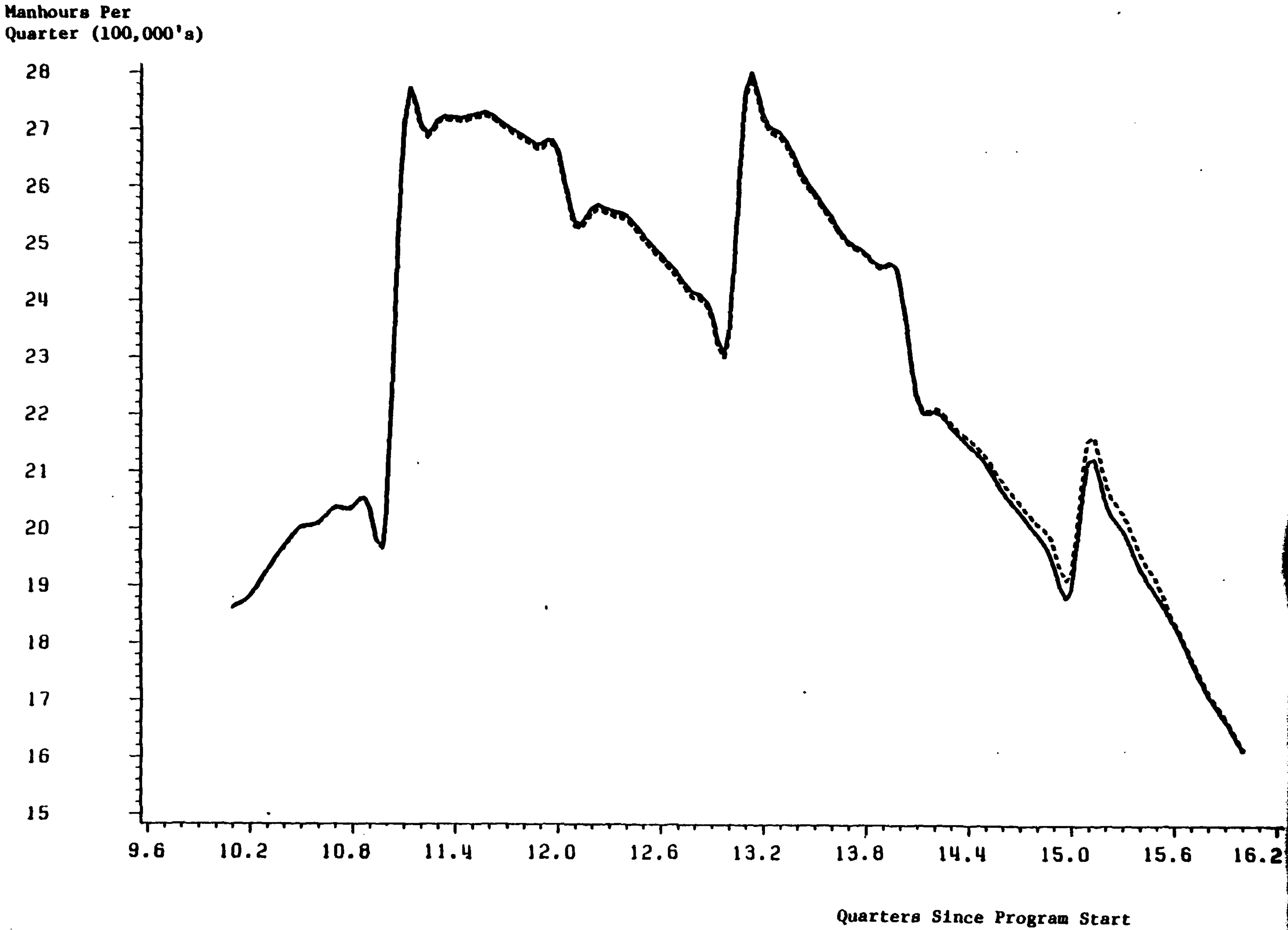

Figure 12. Change Delivery Rate in Quarters 14 and 15

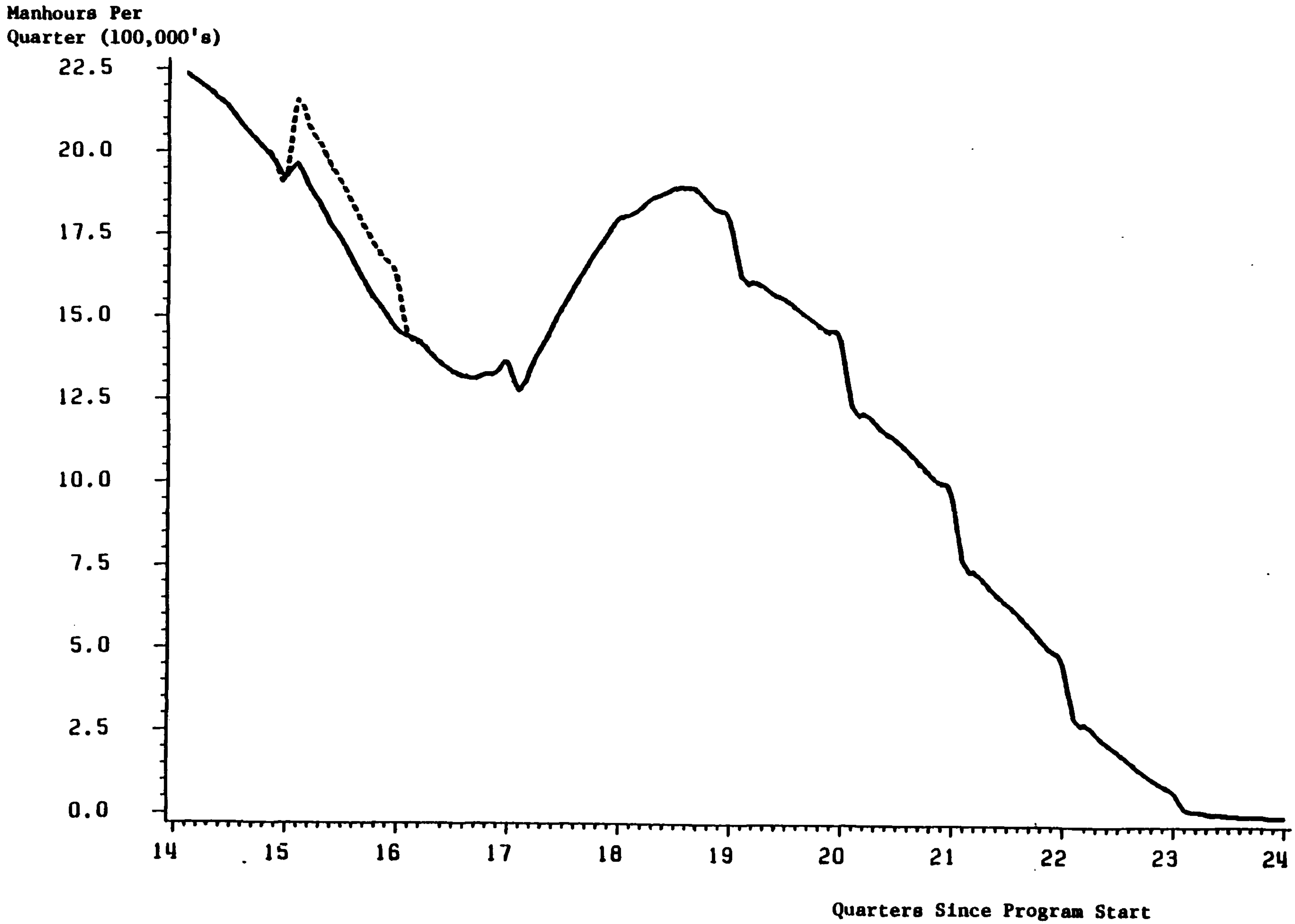

Figure 13. Delay Lot Release Date for the Last Lot by One Quarter

are partly offset by the compressed schedule for the last lot. The effect of these changes is a reduction of the time path in quarter 15 coupled with a very small increase in expenditures starting in quarter 16. The net effect of these changes is to reduce program cost.

Finally we considered the effect of beginning a lot in the middle of the program earlier. In Figure 14 the release date for lot 7 is moved from the beginning of quarter 11 to the beginning of quarter 10. This increases resource use in quarter 10 by permitting work to take place on lot 7. Resource use is also increased for the other lots during quarter 10 because V is higher. Later in the program resource use is decreased due to the lengthening of lot 7. The net effect of these changes is to raise program costs.

E. Summary

Our objective in this study was to provide a model of airframe production that is well grounded in theory, estimated from actual data and sensitive to exogenous delivery schedule effects. In this chapter the rationale for such a model is provided, the functional form is derived, the estimation procedure and the parameter estimates are reported, and the sensitivity of the estimated model to delivery schedule is examined.

The sensitivity analyses clearly imply that some alternative delivery schedules would have resulted in lower costs for the C141 program. If so, we must ask, "Why were these lower cost, higher benefit schedules not chosen?"

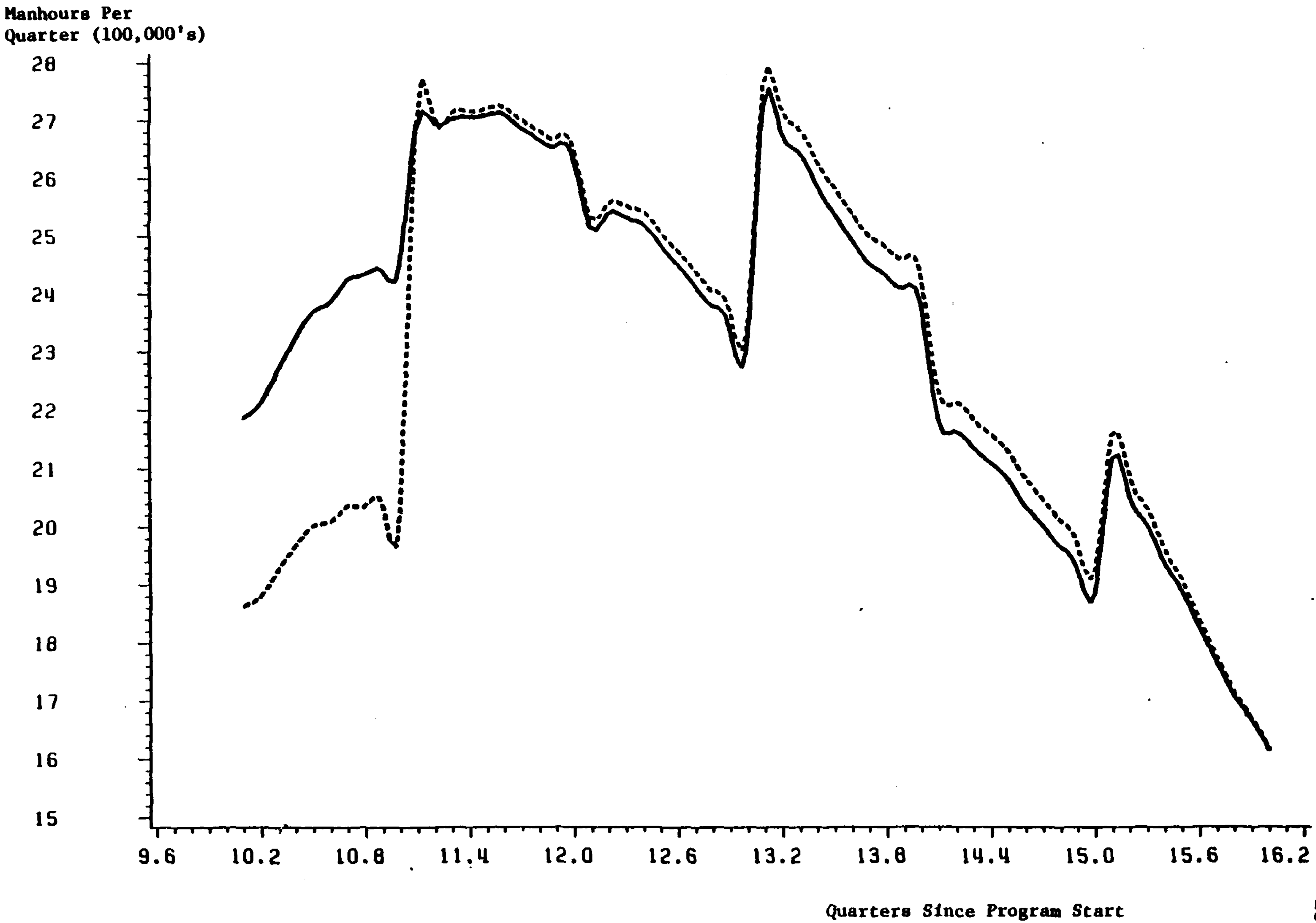

Figure 14. Start Lot 7 One Quarter Earlier

Certainly one possibility is that the decision makers know better than the model what is best. There are several areas in which flaws in the model may be important. One is the lack of hiring and firing costs. A second is the incomplete interaction among the airframes that is permitted in the model. To elaborate, the model does not permit work on an airframe to start later than the lot release date or to end sooner than the delivery date. From the point of view of the single airframe, neither of these events would ever be optimal. If starting late or ending early could affect V, then from the point of view of the program, they may be attractive. As it is now, V is completely determined by the lot release dates and the delivery schedule. Of course, more and better data might permit more accurate and different estimated parameters too.

On the other hand, it is also possible that with a tool which permits decisionmakers to grasp the program implications of funding cuts, stretchouts and of altered delivery schedules, more optimal decisions will be made. Management science is based on possibilities such as these.

VI. ESTIMATION OF THE MODEL EARLY IN THE PROGRAM

A. Introduction

To be useful in program management, a model must not only be accurate, but it must also produce timely answers. This raises a very important question, "When might a program manager expect to be able to get reasonable predictions from a model like the revised C-141 model?" This question is addressed in three different ways in this chapter and the next.

First, the model is estimated using the F-102 data that was described in Chapter IV. This permits us to check on the stability of the estimated coefficients across two very different programs. To the extent that these coefficients are stable, the sensitivity analyses in Chapter V may be generally indicative of resource response to schedule changes across programs. That is, to the extent that the coefficients are stable, the estimated results from the C-141 may provide reasonable predictions without any cost data from the new program.

It should be emphasized that these predictions are not likely to be very accurate, however. They are best viewed as predictions of the direction of changes rather than predictions of the magnitude of changes. The remainder of this chapter is devoted to improving these results.

In section C we investigate combining the revised model with a cost estimating relation to do a better job of estimating the magnitude of the resource effects on a new program.

In section D we investigate the ability of early observations on the C-141 program to estimate the revised model. Here we ask the question, "How many observations on a new program are necessary before that program can be modeled without using prior information?"

In section E the results of sections C and D are generalized to permit combining information from prior programs with early actual data from the new program. These results are summarized and discussed in section F. They are applied to the T-38/F-5 program in Chapter VII.

B. The Revised Model and the F-102 Program

In this section the revised model that was developed in Chapter V is estimated using the F-102 data. This constitutes a validation effort for the revised model.

The data for the model validation was obtained from the "F-102 Program Cost History [8]". The planned delivery sequence, actual delivery sequence and the month of delivery for the airframe in lots four through eleven along with the calculated cost (manhours) per month by lot were used. This data is described in Chapter IV.

The revised model requires cost per unit time (month) for a lot and the delivery dates. The planned delivery sequence number, actual delivery sequence number, and month of delivery are known for each airframe. The delivery month used in the estimation was obtained by matching the planned delivery sequence number with the same actual delivery sequence number. The delivery month of the matching delivery sequence number was used as the month of delivery for the airframe planned as that sequence number. In this manner, the planned delivery schedule was approximated. For a given batch, the airframe deliveries were spread evenly over the month.

The data contained lots four through eleven of the F-102 program because, as previously discussed, the cost per month could be calculated only for those lots. Lots four through eleven contain 207 airframes. Equation (5.8) was estimated for this data using SAS's Proc NLIN [27]. The results of this regression are presented with the C-141 results in Table 22.

Table 22

Parameter Estimates and Asymptotic Standard Errors

	C-141		F-102	
Parameters	Estimates	Standard Errors	Estimates	Standard Errors
β_0	1.150	0.688	3.486	3.335
β_1	3.045	1.162	2.027	1.818
δ	0.484	0.064	0.333	0.416
γ	1.002	.004	1.003	0.017
ν	-0.440	.165	-0.027	0.540
ρ	0.002	.004	0.0008	0.006
MSE	3.66×10^{10}		3.33×10^{10}	

As noted in Chapter IV, the F-102 data is not of very high quality for our purposes. It yields only 57 data points and these are obtained after making use of estimates of the percentage of airframes completed each month. This accounts for the relatively high standard errors reported in Table 22. In spite of this fact, the estimated coefficients are remarkably close for the two programs. This is particularly interesting considering the vast differences in the two airframes.

The scaling factor β_0 is of course different for the two programs. One should not be surprised that a months work on a fighter airframe is of a different scale than a quarters work on a transport airframe. The other coefficients, β_1, δ , and γ are all very close. The corresponding coefficients are easily within one estimated standard error of each other. If the discount rate, ρ, for the F-102 data is multiplied by three (three months to a quarter) the two estimates of ρ are almost exactly the same. Only ν is rather different for the F-102 data; but ν is estimated to have a large standard error relative to its size and it too falls within one standard error of the C-141 estimate.

Figure 15 depicts the time path of monthly resource use predicted for the F-102 program together with the data available on resources used on the program. Figure 15 is quite comparable to the similar Figure 9 for the C-141 program. This further supports the idea that the revised model is rather stable across programs.

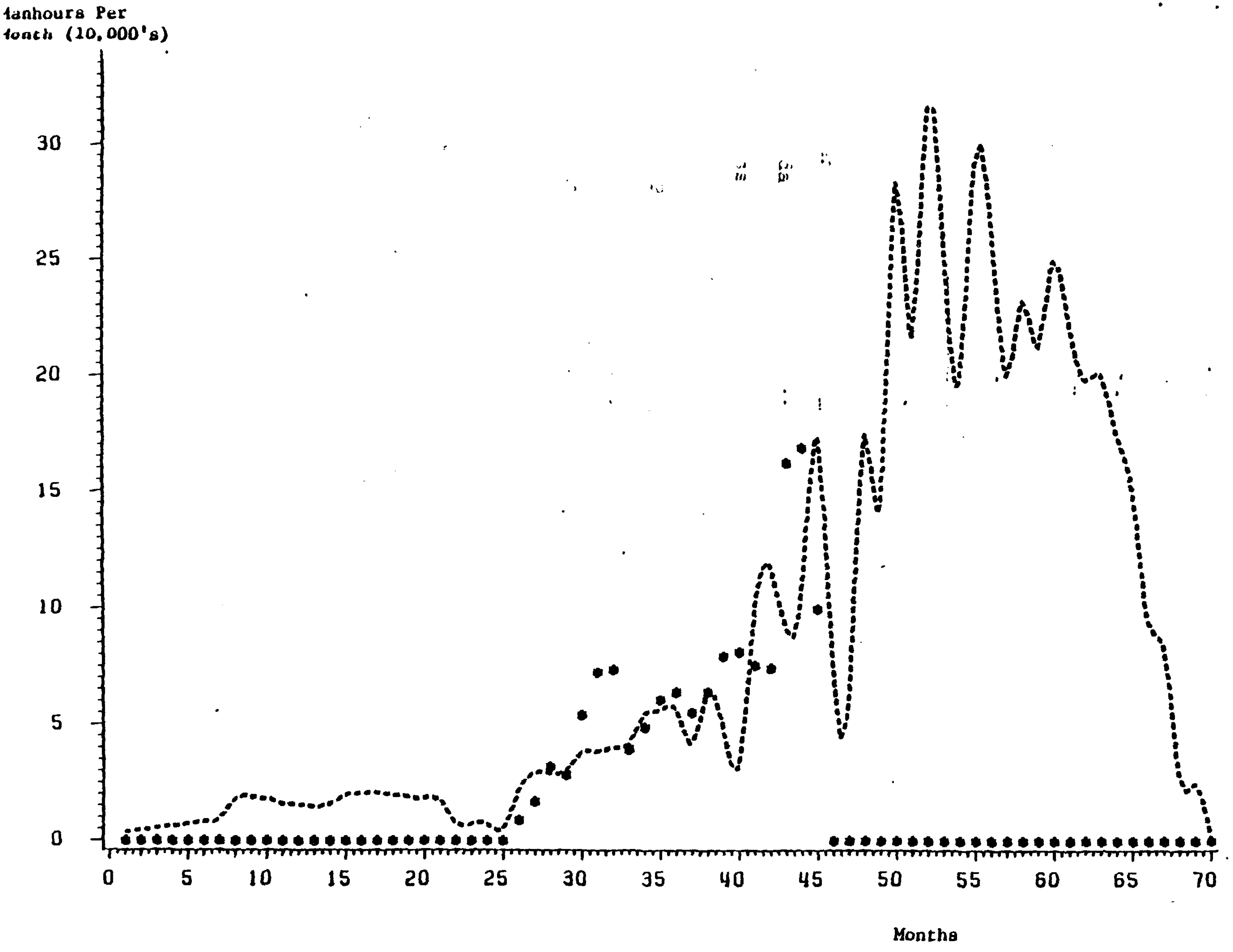

Figure 15. The Time Path of Monthly Resource Use - F-102 Program Estimated from Data on Months 26-45.

The fact that these coefficients are stable over the two programs indicates that the shape of the estimated profile of resource use for the C-141 program is likely to be indicative of the profile for other programs. Furthermore, the direction of changes in the profile associated with changes in the delivery schedule is likely to be the same. However, the large difference in the scaling constant, β_0, for the two programs makes predicting the magnitude of resource use and changes in resource use impossible with the model as it now stands.

A method for solving this prediction problem is investigated in the section below.

C. <u>The Revised Model and Cost Estimating Relations</u>

The prediction problem raised in the last section is not unusual. Prior to the production of any airframe, attempts are made to estimate costs of production. One of the best simple ways to make these predictions is to use cost estimating relations (CER). These cross-sectional relations are concerned with predicting the level of costs as a function of the airframe design. As such, they provide an ideal way to adjust the scaling factor in the revised model for different airframe designs.

Our recommended procedure is as follows. First, estimate the labor hours for some number of airframes in the new program using a CER. For example, Large, Campbell, and Cates [15 , p.82] provide a cost estimating relation for the recurring manufacturing labor hours of the first 25 airframes as:

$$ML_{25} = 2.08\ W^{.799} S^{.414} T^{-.633} \tag{6.1}$$

where ML_{25} = recurring manufacturing man-hours for the first 25 airframes (thousands),

W = airframe unit weight (lb.),

S = maximum speed (kn.),

T = number of quarters after 1942 that first flight of a production aircraft occurred.

Let the labor hours predicted by (6.1) be $\hat{ML}_{25}$. But the revised model can also be used to predict the labor hours required to produce the first 25 airframes as:

$$\hat{Y}_{25} = \sum_{i=1}^{25} \hat{X}_i(t_{di}) - \hat{X}_i(t_{si}) \tag{6.2}$$

where $X_i(t_{di})-X_i(t_{si})$ is given by (5.6). These two predictions of the labor required to produce the first 25 airframes can be used to adjust the scaling factor in the revised model as:

$$\hat{\beta}_{0a} = \hat{\beta}_0 \cdot \hat{ML}_{25}/\hat{Y}_{25} \tag{6.3}$$

This has the effect of shifting the revised model by a constant amount per airframe so that the revised model goes through the point predicted by Rand's CER. This procedure is demonstrated for the F-5/T-38 program in Chapter VII.

D. <u>Estimating with Early Actual Data</u>

We also investigated the properties of the revised model when estimated using only the early actual data from the C-141 program. Here we sequentially estimated the model using progressively more quarters of observations on the C-141 program. Table 23 summarizes the results from five of these runs. The first and second columns of Table 23 list the quarters of data and the number of observations used to estimate the model. The mean squared error is given in the third column while the parameter estimates are given in columns five through nine.

Table 23

Sequential Estimation of the Revised C-141 Model

Qtrs	Obs	MSE	β_0	β_1	δ	γ	ν	ρ
8	23	0.62	4.27	3.72	.20	1.0006	0	.0009
9	28	0.77	4.79	3.48	.32	1.0006	0	.0008
10	33	0.92	4.98	3.69	.37	1.0009	0	.001
11	37	0.96	4.06	3.20	.44	1.01	-.09	.02
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
24	89	3.66	1.15	3.04	.48	1.002	-.44	.002

At first glance Table 23 appears to be discouraging. The MSE for the early observations seems to be too low in terms of what will eventually happen. Also the values estimated for β_0 and for ν from the early observations are very different than those of the full model. Furthermore, these parameter estimates show no sign of getting closer to their final values any time soon. (In fact, the results for the next set of five quarters show similar results.) These results are rather different from a similar analysis done on the earlier version of the C-141 model where parameter estimates tended to be rather stable from the early models to the latter models.

In fact the situation is not nearly as bad as it appears to be. The parameter estimates for β_0 and ν are consistently strongly correlated. This indicates that models with high values for β_0 and ν (like the 8 quarter model) cannot be easily distinguished from models with low values of β_0 and ν (like the 24 quarter model). Table 24 illustrates this point.

Table 24

Comparison of Predictive Capability

Qrts. in limited model	Obs. in limited model	N	SSD	Corr.
8	23	66	928.6	.97
9	28	61	59.9	.99
10	33	56	41.9	.99
11	37	52	61.5	.95
12	43	46	29.9	.88

In addition to the stability of parameter estimates from early data, we were concerned with the ability of the limited data model to estimate the final model. To evaluate this ability we compare the cost predictions from the limited data models with those for future quarters from the full model. Table 24 shows the results of this comparison of the limited data models with the full model. The first two columns give the number of quarters of data and the number of observations used in the limited data model. The third column gives N, the number of pairs of predicted future observations that were considered. The fourth column lists the sum of squared differences between the predicted values generated by the two models. The fifth column lists the estimated correlation coefficient between β_0 and ν for the corresponding limited model.

While the sum of squared differences decline as the number of observations increase, the number of points being predicted do too. In fact, it seems that the model estimated from limited data is very close to the final model after the eighth quarter. The mean squared error between the two models is only about one fourth of the mean squared error between the final model and the data.

Therefore, even though the models estimated with limited data have coefficients which differ in magnitude from the final model, the predictions of the models are almost the same. We conclude that after the eighth quarter the limited data model does a reasonable job of predicting the final cost profile of the program.

Table 23 points out one additional problem with the limited data model. As more data is added MSE, the estimate of variance, increases. Nevertheless, by quarter 12 it is only one fourth the MSE of the final model. This indicates a heteroscedasticity problem with the model. (Its variance seems to increase with time). This problem can be easily dealt with when estimating the model from early actual data, but it must not be ignored.

E. A Bayesian Updating Procedure

In this section we consider a Bayesian procedure for combining the revised C-141 model with early actual observations on a new program. This can be used to estimate the cost profile for the remainder of the new program. To do this we treat the revised C-141 model (after adjusting the scaling factor) as a natural conjugate prior distribution for the new program. (See Judge et. al. [14, pp. 97-104] for a derivation of the procedure.

Let

$\hat{\beta}_C$ = the vector of parameters estimates for the C-141 model (including the adjusted scaling factor)

$\hat{\beta}_N$ = the vector of parameter estimates from early actual data for the new program

$\hat{\beta}_u$ = the vector of updated parameter estimates

Q_C = the inverse of the estimated variance-covariance matrix of parameter estimates for the C-141 model

Q_N = the inverse of the estimated variance-covariance matrix of parameter estimates from early actual data on the new program.

The Bayesian updating procedure defines $\hat{\beta}_u$ as

$$\hat{\beta}_u = (Q_N + Q_C)^{-1} (Q_N \hat{\beta}_N + Q_C\hat{\beta}_C) \tag{6.4}$$

While the procedure is based on a rather detailed derivation, this result is straightforward and intuitive; it defines the updated parameter estimate as the weighted average of the two estimates. The weights are the inverses of the estimated variance-covariance matrices. In applying this result to the F-5/T-38 program in Chapter VII we make use of the slightly more general form of (6.4) given in equation (6.5).

$$\hat{\beta}_u = (Q_N + kQ_C)^{-1}(Q_N\hat{\beta}_N + kQ_C\hat{\beta}_C) \tag{6.5}$$

The extra factor in (6.5) is intended to help compensate for two problems. One is the heteroscedasticity problem; the other is the fact that the scaling factor has been adjusted for design differences between the aircraft.

F. <u>Summary</u>

In this chapter we have investigated the application of the revised C-141 model to other airframe programs. We first noticed that the estimated coefficients for the C-141 data and the F-102 data were very similar. With the exception of the scaling factor they fall within one standard deviation of each other. This implies a great deal of commonality between the two models.

Next we investigated a method to adjust the scaling factor to the new program. A procedure for using CER to do this was reported.

We then investigated the use of early actual data to estimate the revised model. This investigation showed that the early actual data do a reasonable job of forecasting the cost profile of the entire program.

Finally, we reported a procedure to combine the C-141 model with early actual data on a new program.

In this chapter we have reported two good ways to use the revised model to predict a cost profile for a new program. The first way, adjusting the scaling factor with a CER and using the parameter estimates from the C-141 program, requires <u>no cost data</u> from the new program. It can be used for program management very early in the program. The second method, using early actual data, does require some program cost information. Furthermore, we have reported a way to combine the two technique which offers the hope of even better performance. These methods are demonstrated using the *T*-38 data in Chapter VII.

VII. APPLICATION OF THE MODEL TO THE F-5/T-38 PROGRAM

A. Introduction

In this chapter the statistical methods developed in Chapter VI are applied to data from the F-5/T-38 program. The contractor, Northrop Aircraft Corporation, provided data on forty-eight lots of aircraft covering the T-38 and seven models of the F-5 aircraft. We are primarily interested in the ability of the model to predict a cost profile early in the program. The first fourteen lots of the program consist entirely of T-38 aircraft, therefore, we confined our attention to the observations on the T-38. This avoids having to adjust our estimators for the several designs of F-5 aircraft.

This chapter is organized as follows. In the next section the T-38 data is discussed and the model is modified to conform to the data. In section C the ability of the C-141 model to fit the data is investigated. This is done by adjusting the scaling factor as discussed in Chapter VI. The estimation of the model from early actual data is described in section D and the Bayesian updating procedure is applied to the data in section E. In section F our efforts to use the C-141 model to predict a cost profile for the T-38/F-5 program are summarized and conclusions are drawn.

B. The T-38 Data and Model

The T-38 data for this study, with the exception of the delivery dates, was obtained from the manufacturer. Total labor hours per lot, the number of airframes per lot, DCPR weight and starting dates for each of the thirty-seven lots of the T-38 program were given. Delivery dates were obtained from the OASD (PA&E) [20] "Acceptance Rates and Tooling Capacity for Selected Military Aircraft." Some data adjustments were required. Because the acceptance dates were given by month and year only, the actual acceptance date was assigned by spreading the aircraft accepted in a month evenly over the month of their acceptance. In addition, the starting dates for the first five lots were given only by month and year. So, to minimize possible error, these were also assumed to have been the fifteenth day of the given month.

Preliminary analysis of the data revealed that the fluctuation of DCPR weight among T-38's was insignificant so that it was not needed as a weighting factor. The T-38 data base used in the estimation contains 37 observations (T-38 lots) and a total of 1187 airframes, it is included as Appendix C.

Since the T-38 data is lot data instead of data on lots by time period, the C-141 model must be modified for application to the data. The equation estimated with the C-141 data, equation (5.8) is reproduced as (7.1)

$$\sum_{i=K_j}^{n_j} X_i(T_2) - X_i(T_1) = \sum_{i=K_j}^{n_j} \beta_0 (i-1/2)^{-\gamma\delta} \Gamma^{-\gamma}[\rho(t_{di}-t_{si})/(\gamma-1),\ \beta_1]$$

$$V^{-\gamma\nu}(T_1,T_2)\{\Gamma[\gamma\rho(t_{di}-T_1)/(\gamma-1)] \qquad (7.1)$$

$$-\Gamma\ [\gamma\rho(t_{di}-T_2)/(\gamma-1)\beta_1]\}$$

In this case, the period of observation, T_1 to T_2 is the entire period from the start of work to the end of work on airframe i, i.e. t_{si} to t_{di}. Replacing T_2 and T_1 with t_{di} and t_{si} caused one of the terms in (7.1) to go to zero. Thus the basic equation for the T-38 data becomes:

$$\sum_{i=K_j}^{n_j} X_i(t_{di}) - X_i(t_{si}) = \sum_{i=K_j}^{n_j} \beta_0 (i-1/2)^{-\gamma\delta} \Gamma^{-\gamma} [\rho(t_{di}-t_{si})/(\gamma-1), \beta_1] \\ V^{-\gamma\nu} \Gamma[\gamma\rho(t_{di}-t_{si})/(\gamma-1),\ \beta_1] \tag{7.2}$$

Equation (7.2) is the basis for the comparison of the C-141 model to the T-38 data throughout this chapter.

C. Adjusting the Scaling Factor

Large, Campbell and Cates [15] give the airframe unit weight, the speed and date of first flight for the T-38 aircraft. Using this information the recurring manufacturing hours for the first 25 airframes in the T-38 program can be estimated with (6.1) as:

$$\hat{ML}_{25} = 2.08\ (5376)^{.799}\ (750)^{.414}\ (64)^{-.633} \tag{7.3}$$

$$= 2215 \text{ in thousands of man-hours}$$

or $= 22.15$ in hundreds of thousands of man-hours.

Likewise, using equation (7.2) with k_j=1 and n_j=25 an estimate of the manhours for the first 25 airframes from the C-141 model is given as:

$$\hat{Y}_{25} = \sum_{i=1}^{25} 1.15\ (i-1/2)^{-.485}\ \Gamma^{-1.002}[1.0(t_{di}-t_{si}),3.045] \\ V^{-.44}\Gamma[1.002(t_{di}-t_{si}),3.045] \tag{7.4}$$

$$= 55.66$$

Using these estimates in (6.3) gives the adjusted scaling factor as

$$\hat{\beta}_{oa} = (1.15)(22.15)/55.66 = .4576 \tag{7.5}$$

The data for the T-38 (the left side of (7.2)) are divided by $n_j - k_j + 1$ to form the manhours per airframe. Likewise, the right side of (7.2) evaluated at the parameter estimates from the C-141 with β_{oa} replacing $\hat{\beta}_0$ is divided by $n_j - k_j + 1$. These two sets of values are plotted verses k_j in Figure 16.

In Figure 16 it is clear that the modified C-141 model overpredicts the resources required for the T-38 until late in the program. Largely this is due to the poor estimate of the labor hours for the first 25 airframes from the RAND CER. If the CER estimate is replaced by the actual cost of the first 25 airframes $\hat{\beta}_{oa}$ becomes 0.2823. This results in Figure 17; a much closer fit to the T-38 data.

The details of these two figures are more easily seen in Figures 18 and 19 which consist of just the first nine lots of the T-38 program. In both cases the estimated relations anticipate the cost rise in the fifth lot. The model assigns these costs to earlier airframes than Northrup's accountants do, however. Again it should be emphasized that Figures 16 and 18 are based on a model which uses no cost data from the F-5/T-38 program. It could be used well before the program starts.

D. Estimation from Early Actual Data

We also investigated the ability of the revised model to estimate early observations on the T-38 program. Here we sequentially estimated the model using progressively more lots of the T-38 data. Table 25 summarizes the results from seven of these runs. The first and second columns of Table 25

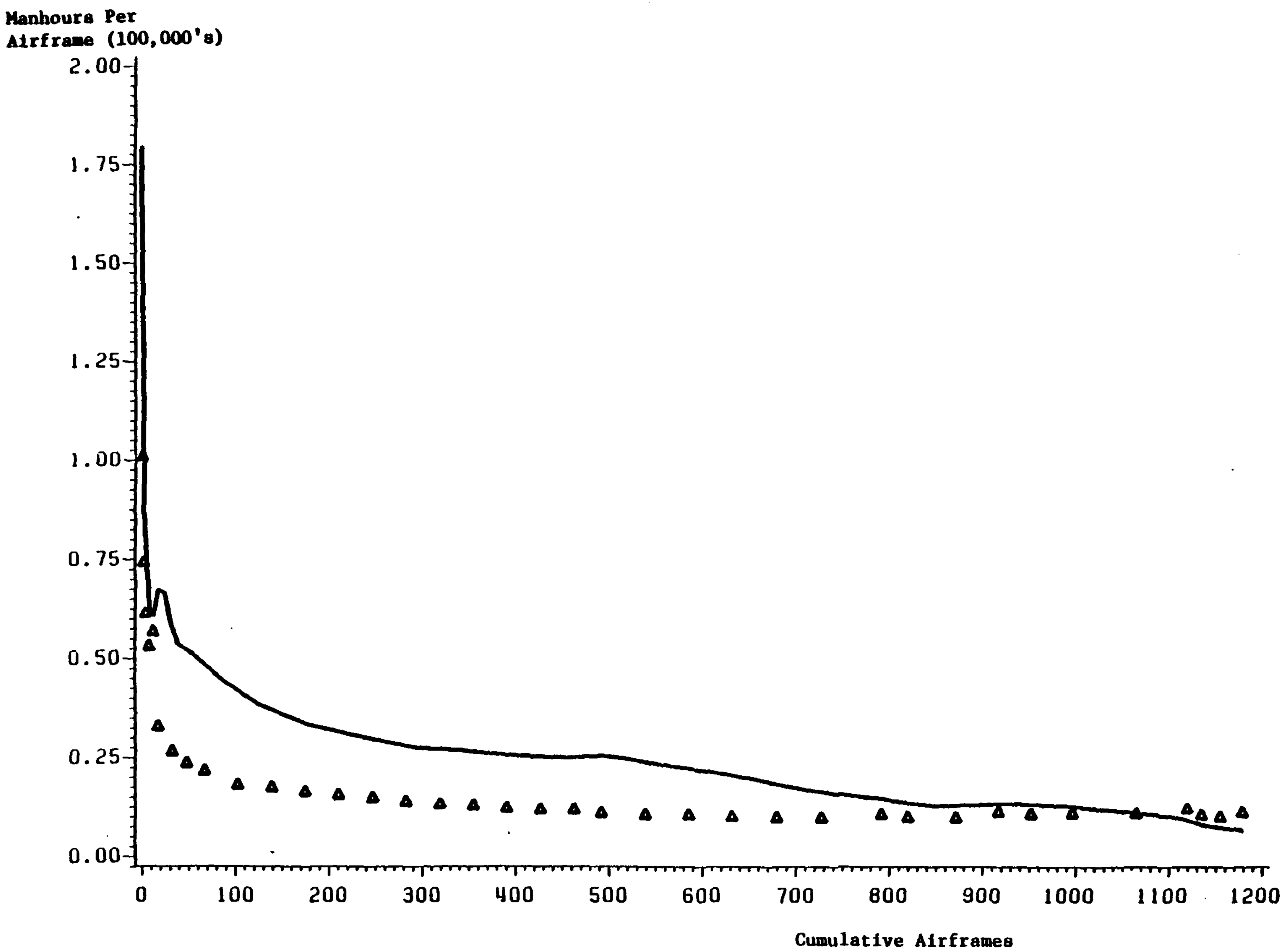

Figure 16. The C-141 Model and T-38 Data using a CER

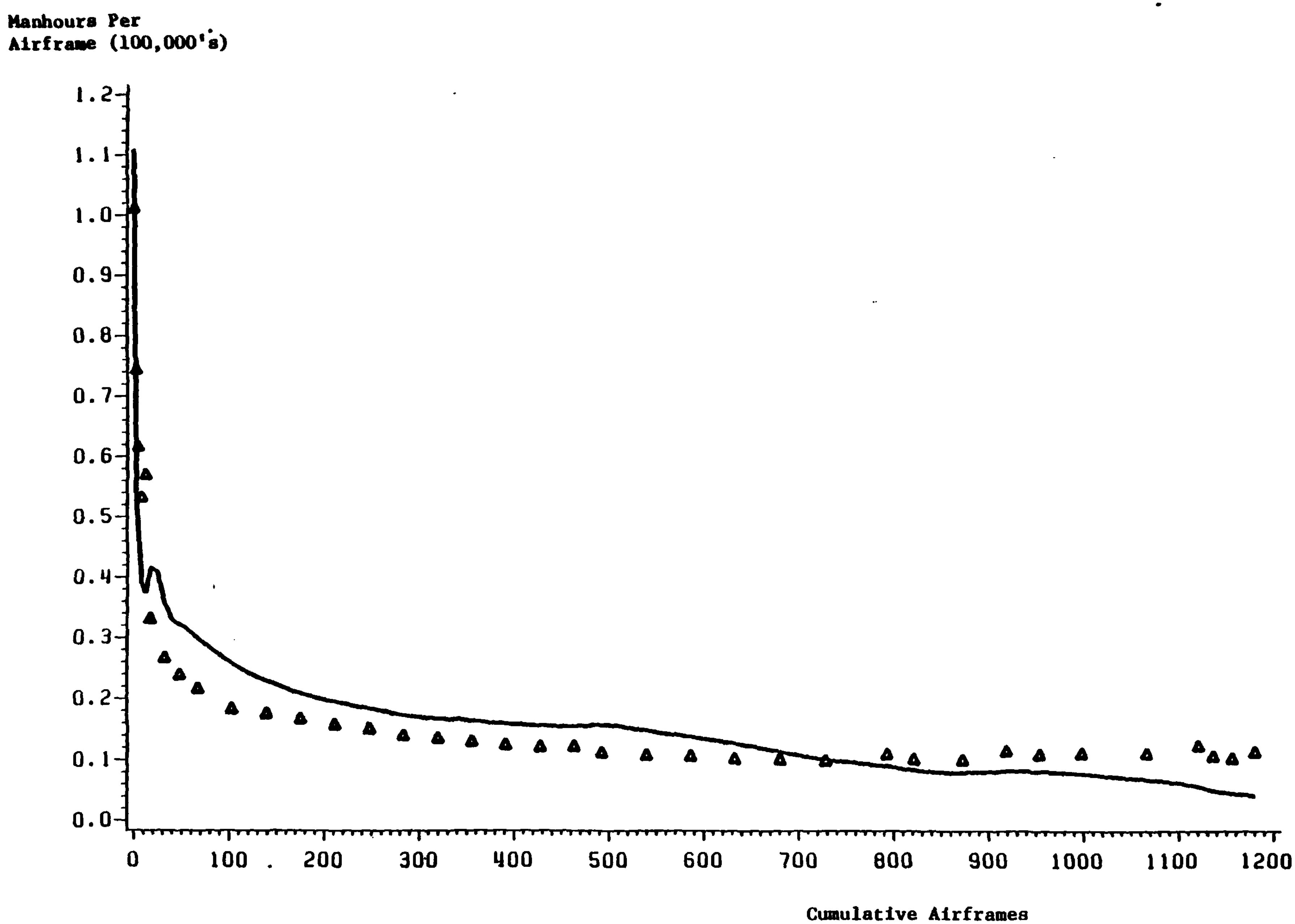

Figure 17. The C-141 Model and T-38 Data Using the Actual Cost of the First 25 Airframes.

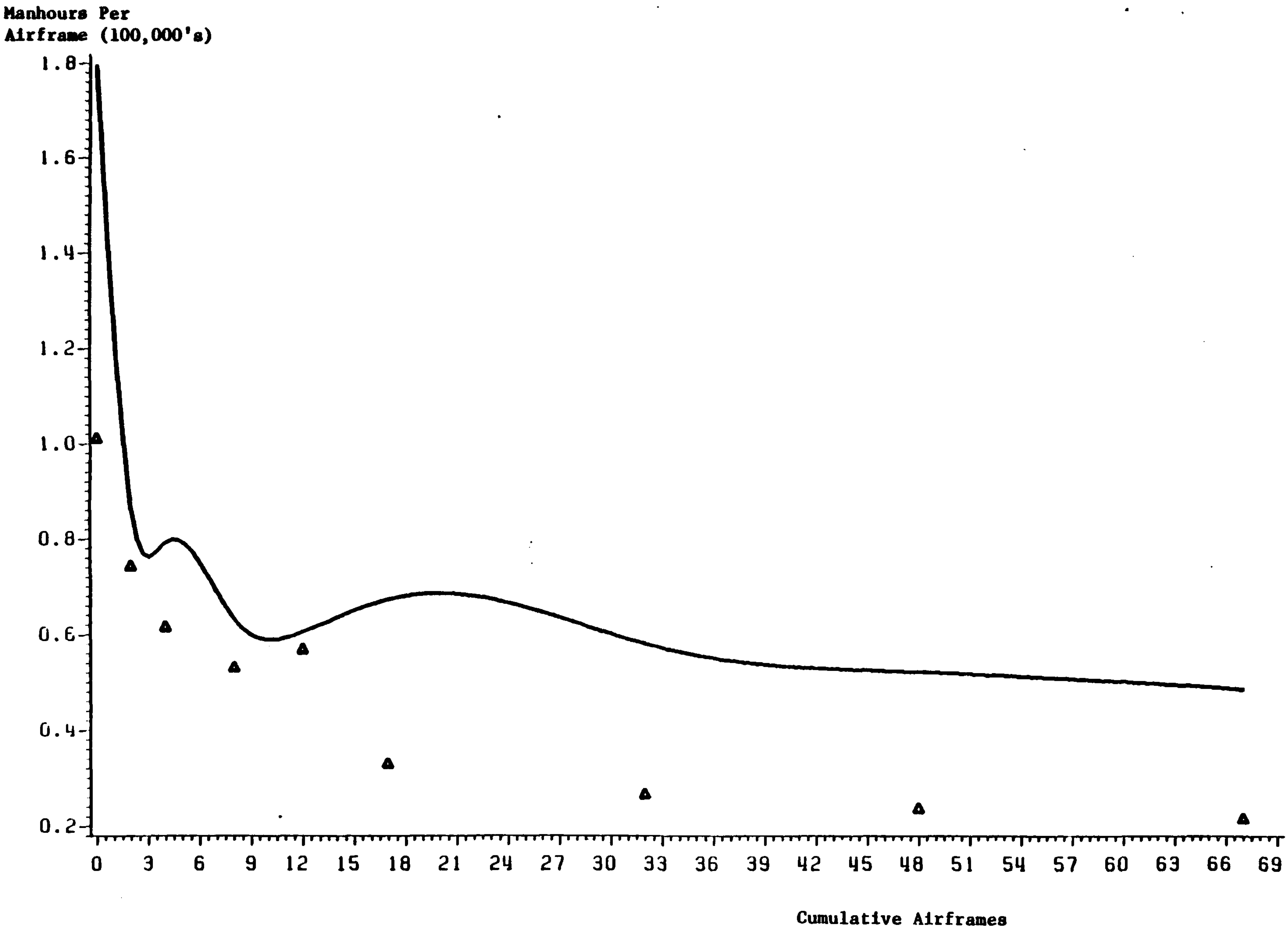

Figure 18. The C-141 Model and T-38 Data Using a CER

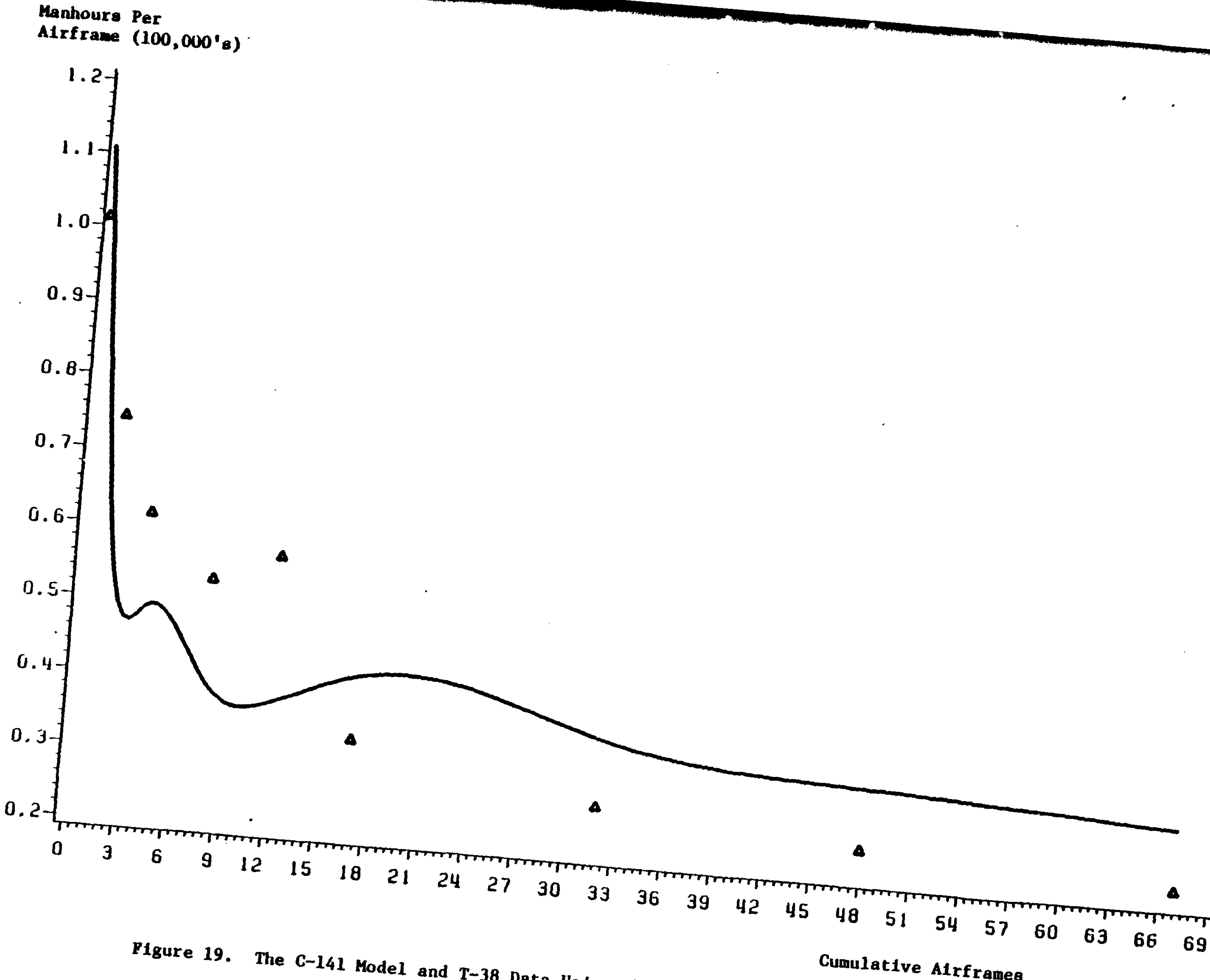

Figure 19. The C-141 Model and T-38 Data Using the Actual Cost of the First 25 Airframes.

list the quarter when the data was available and the number of observations used to estimate the model. The mean squared error is given in the third column while the parameter estimates are given in columns four through nine.

Comparing Table 25 and Table 23 shows some dramatic differences between the T-38 lot data and the C-141 quarterly data.

At the eight quarter point, two years after production started, the C-141 program had 23 observations available for use. At the same point in time the T-38 data yields only 2 observations. In addition to the fewer observations, the T-38 data does not capture the time dimension of the production problem. This lot data is aggregated over all the months of a lot's production. As a result it exhibits very little variation in the time dimension. The effects of this show up in Table 25. While the model explains the T-38 data very well (MSE is substantially smaller than for the C-141 data) the parameters of the model are not estimated reliably. The variance covariance matrix is almost singular, the pairs ($\hat{\delta}$ and $\hat{\gamma}$) and ($\hat{\beta}$ and $\hat{\rho}$) have very high correlations.

The message is clear, the revised model is substantially more general than necessary to explain the T-38 lot data. This is not surprising. The model was designed to provide a production cost profile for a program. The T-38 data merely records the cost of airframes by lots, it tells us very little about the time dimension of the problem. As a result it should

Table 25

Sequential Estimation of the Revised Model from T-38 Lot Data

Qrts.	Obs.	MSE	β_0	β_1	δ	γ	ρ	ν
15	9	0.32	6.07	.0004	.30	1.22	.01	0
16	10	0.24	5.92	.0005	.30	1.22	.01	0
17	11	0.20	5.87	.0005	.30	1.22	.01	0
18	12	0.18	5.80	.0005	.29	1.22	.01	0
19	13	0.16	5.86	.0005	.29	1.22	.01	0
20	14	0.25	5.83	.0004	.29	1.22	.01	0
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
35	37	0.29	5.05	.0003	.23	1.24	.01	0

estimate parameters associated with the time dimension reliability.

The parameter estimates in Table 25 are of course substantially different from those of Table 23. Despite this fact the parameters of Table 23 do fit the T-38 data rather well.

The main lesson to be learned from this exercise is that lot data do not provide a reliable estimate of a program's cost profile.

E. The Bayesian Updating Procedure

The updating procedure discussed in Chapter VI is applied to the situation where nine data points of the T-38 program are available. Using the notation of Chapter VI

$$\hat{\beta}_C = [0.4576 \quad .4842 \quad .30445 \quad 1.0017 \quad .002 \quad .4402]$$

$$\hat{\beta}_N = [5.07 \quad .2979 \quad .37\text{E-}4 \quad 1.2179 \quad .01 \quad 0]$$

$$Q_C = \begin{matrix} 1.2427 & 23.657 & .74658 & 724.04 & -702.66 & -8.9298 \\ -107.77 & 949.01 & -9.0515 & -.56438 & 3262.5 & -671.83 \\ 515.85 & -2602.8 & 32.134 & -19951 & 1080.6 & 2809.5 \\ 762353 & -3850384 & 39795 & -25916869 & 1679256 & 4155073 \\ -642353 & 3238857 & -35011 & 22631886 & -1393377 & -3495291 \\ -21.007 & 105.29 & 1.9159 & 5269.5 & -3855.1 & -114.54 \end{matrix},$$

and

$$Q_N = \begin{matrix} -1.2182\text{E-}7 & -1.221\text{E-}5 & -2.3021\text{E-}4 & 2.9597\text{E-}6 & -1.5182\text{E-}7 & 8.625\text{E-}5 \\ -1.221\text{E-}5 & -4.1186\text{E-}7 & -.024607 & 5.8376\text{E-}4 & 3.7724\text{E-}7 & -1.5081\text{E-} \\ -2.3021\text{E-}4 & -.024607 & -.2324 & .0059522 & -.0024718 & .95271 \\ 2.9597\text{E-}6 & 5.8376\text{E-}4 & .0059522 & 2.125\text{E-}9 & 2.9877\text{E-}8 & -2.5018\text{E-} \\ -1.5182\text{E-}7 & 3.7724\text{E-}7 & -.0024718 & 2.9877\text{E-}8 & 2.4321\text{E-}5 & -.0086448 \\ 8.625\text{E-}5 & -1.5081\text{E-}4 & .95271 & -2.5018\text{E-}4 & -.0086448 & 7.3855 \end{matrix}$$

Applying equation (6.5) with K=1 yields

$$\hat{\beta}_u = [1.0957 \quad .4368 \quad 3.9624 \quad 1.0034 \quad .0063 \quad .2793]$$

In Figure 20 the T-38 data is plotted with the model evaluated at $\hat{\beta}_R$. Figure 20 is very much like Figure 16. This implies that the influence of the CER errors on $\hat{\beta}_C$ is still substantial with K=1. Figure 21 displays the same information with the model evaluated at $\hat{\beta}_N$. Clearly in this case we are better off using the early actual data alone rather than using it to update the C-141 coefficients.

The process of attempting to illustrate the technique using various values of K and different numbers of early actual observations yielded several disturbing conclusions.

First it was found that for some sets of observations the variance covariance matrix was so nearly singular that Q_N could not be computed. It was also found, in some cases, that with $\hat{\beta}_C$ and $\hat{\beta}_N$ both within the usual limits $\hat{\beta}_R$ contained elements that were not believable, i.e. $\hat{\gamma} < 1$ and $\hat{\rho} < 0$. Furthermore in cases where $\hat{\beta}_R$ was believable for K=1, it became improper as K increased. Because of these unstable results, we do not recommend the Bayesian procedure as developed in Chapter VI. One problem with that procedure is the fact that the variance covariance matrices are calculated based on the assumption that the model is nearly linear in the vicinity of the estimated parameters. In this case we are dealing with a highly non-linear model. To get around this problem we decided to combine the two sets of data, the C-141 data and the early actual observations on the T-38 program. The T-38 observations

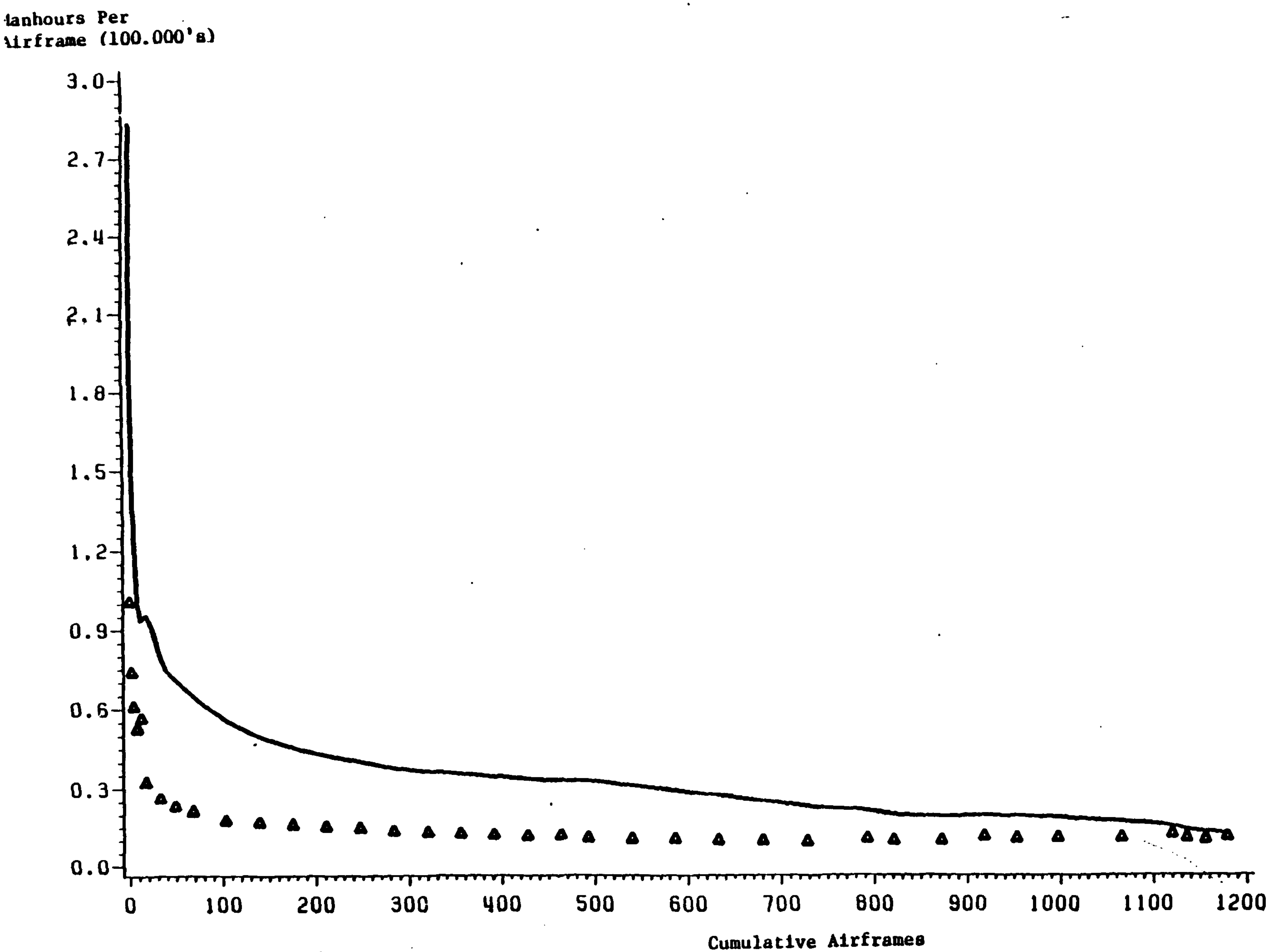

Figure 20. The Model Evaluated at $\hat{\beta}_R$ Using 9 T-38 Observations.

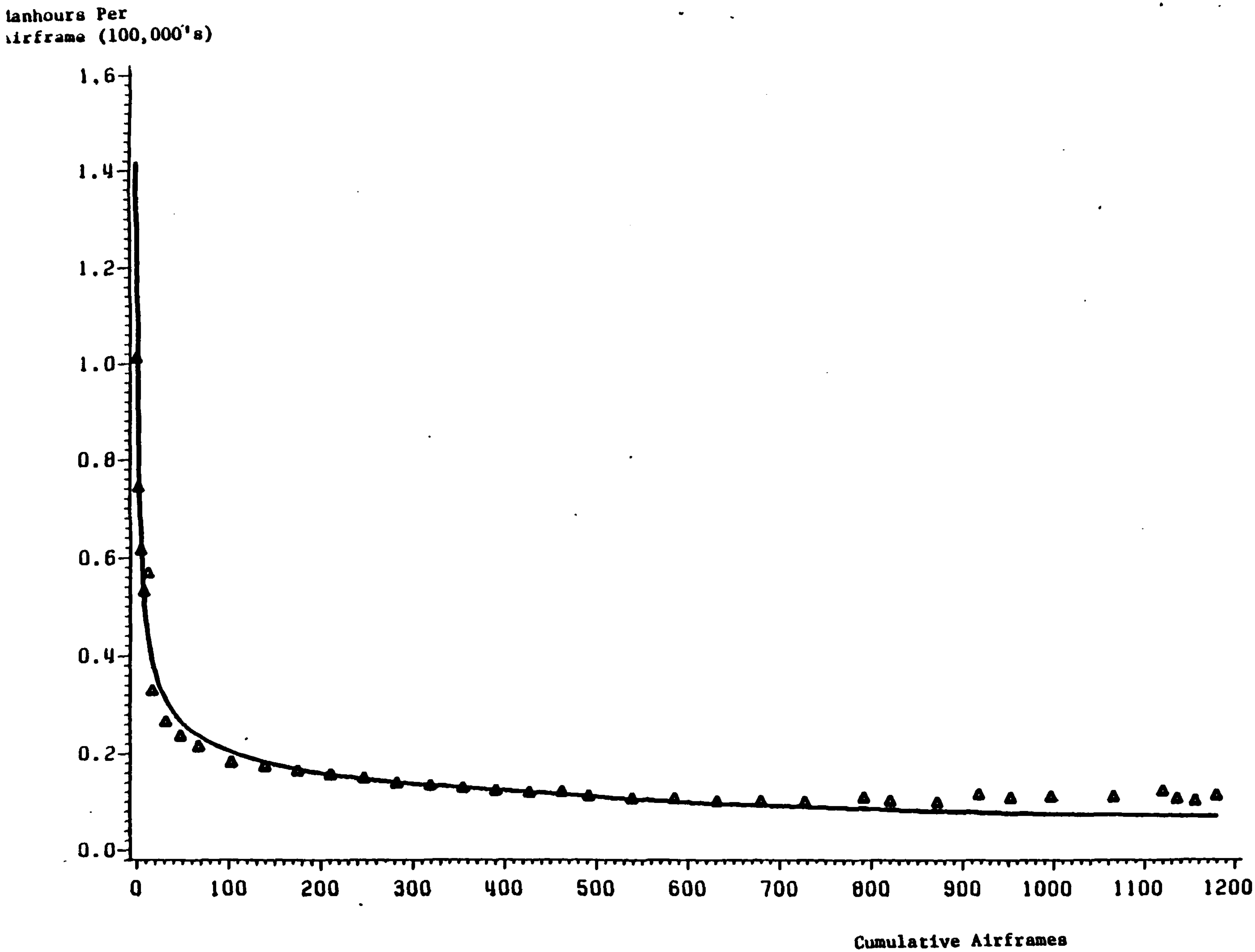

Figure 21. The Model Evaluated at $\hat{\beta}_N$ Using 9 T-38 Observations

were weighted more heavily than the C-141 observations and the C-141 data were adjusted for the scaling factor as described in Chapter VI. This approach did prove to be a useful way to combine the two kinds of information, and it too is a Bayesian procedure. Table 26 presents some representative results from this procedure. Figure 22 compares one of these estimates to the data.

Despite the reasonable results presented here this Bayesian updating procedure has not been adequately tested by the T-38 data. This lot data is just too aggregated to exercise the procedure. That is, the data confronts the model with a prediction problem that has hardly any time dimension, thus many of the model features do not get used.

F. Summary

In this chapter the statistical procedures that were developed in Chapter VI were applied to the problem of using the revised model to predict costs early in the T-38 program.

Observations on the T-38 are not available by time period but only by lot. This data situation has both advantages and disadvantages for testing the model. The advantage is that the model is demonstrated to be useful in describing data that are very different from those data used to estimate it. The disadvantage is that several features of the model are not exercised by the T-38 data set.

Nevertheless, the revised model performed quite well in these circumstances. We conclude that it is applicable to many other airframe cost estimation situations.

Table 26

Weighted Bayesian Estimates for the T-38 Data

N	W	β_0	β_1	δ	G	ρ	ν
1	90	.239	3.07	.481	1.008	.0105	.576
2	45	.370	3.04	.411	1.005	.0055	.417
3	30	.410	.772	.380	1.0001	.00001	.377
4	22	.690	2.20	.519	1.007	.0005	.618
5	18	.69	2.20	.519	1.007	.005	.618
6	15	.43	2.006	.398	1.0001	.0001	.301
7	13	.93	2.52	.287	1.0001	.0001	.000+
8	11	.82	1.61	.400	1.0001	.0001	.102
9	10	.82	1.61	.398	1.0001	.0001	.100

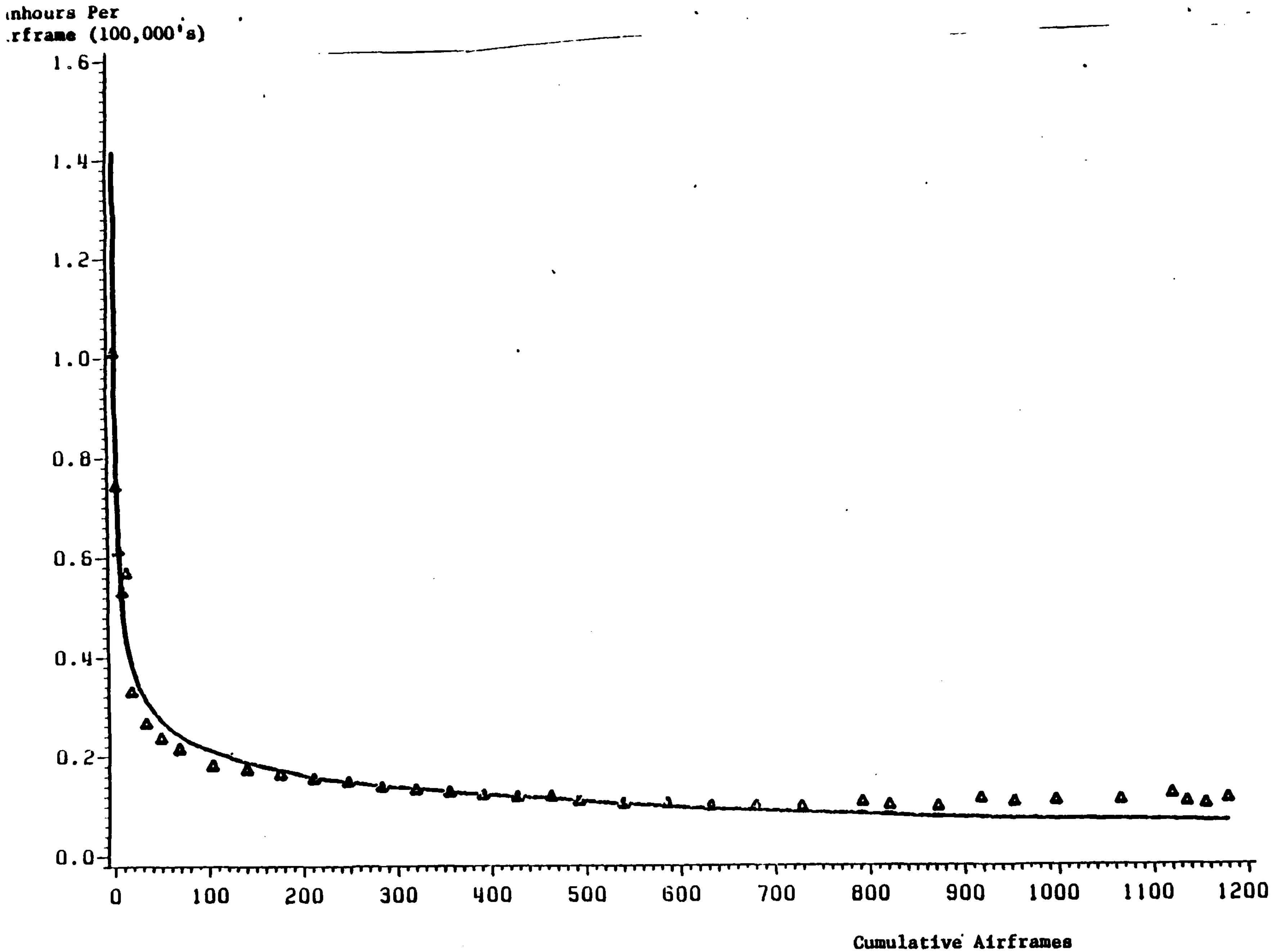

Figure 22. The Model Evaluated with the Weighted Bayesian Estimator with N=9 and W=10

VIII. RESULTS AND AIR FORCE APPLICATIONS

A. Introduction

The purpose of this chapter is to review the results of this research with an emphasis on how the results can be applied by the Air Force.

Our objective for this research effort was to develop, test, and illustrate the use of a significantly new approach to estimating the cost of an airframe production program. The theory was developed to unify previously separate methods of describing program costs. The effort was to result in a cost function that could be estimated from already collected data on Air Force airframes. We were to provide the Air Force with a calibrated tool capable of providing timely answers to significant problems of program management. These objectives have been met by the revised model. It's development is described in Chapter V. The resulting cost function was estimated from C-141 data as reported in Chapter V. It was also applied to the F-102 data as reported in Chapter VI. Also in Chapter VI methods were developed for using the model to estimate airframe costs early in a production program, and in Chapter VII these methods were applied to the T-38 data.

In the sections of this chapter these results are reviewed for application to Air Force program management problems. Section B is titled "Understanding Production Scheduling". There the role of the model in explaining the scheduling problem is discussed. In section C the use of the model by SPO personnel is discussed. Here the ability of the model to shed light on program management problems is discussed. In section D cost estimating techniques are described, and their use at different phases in the acquisition cycle is discussed. Finally, in section E, the role of these results as a basis for further research is discussed.

B. Understanding Production Scheduling

All too often both the critics and the friends of a particular Air Force program argue as if there are simple, almost trivial, relations between production schedules and program costs. Often the learning curve is the only relation used to discuss the implications of alternative production schedules. Likewise, when the timing of foreign military sales is considered only the learning curve is used to analyze the situation. The learning effect is the only production cost driver that is included in the Selected Acquisition Reports (SAR) to Congress on major programs.

Even without explicitly using the model, the four production cost drivers discussed in Chapter V can be used to paint a word picture of the consequences of altering a program's delivery schedule. This could significantly increase the Air Force's ability to communicate internally with higher authority.

For example, consider the first of the sensitivity analyses depicted in Figure 10 of Chapter V. There the first airframe's delivery was to be delayed one month. If a learning curve were used to analyze the situation, there would have been no affect reported. In fact, the effect on total program costs is not large. However, the effect is significant in terms of the the timing of costs and the resources required for other airframes in the program. Delaying the delivery of the first airframe provides more time for learning on airframe one (effect 2), and it reduces the planned speed of the production line (effect 3) both these effects tend to lower cost. They are offset by the fact that work on airframe one is delayed to a time when there are more airframes scheduled to be in the facility. This increases V and the length of the production line (effect 4). Thus the cost of producing all the airframes in the facility is increased during the month that is added to airframe one's delivery date. While these effects are not large, they do amount to almost 300,000 man hours. So they are not trivial.

This research has also contributed to the understanding of production scheduling by unifying earlier models of Smith, Alchian, and Rand. This is also of significance to the Air Force. Without a generalized model, debate about the relations among production rate, delivery rate and program costs boil down to a matter of opinion. With the generalized model there will still be debate and opinion, but there are also explicit statistical tests that can be performed. The model provides a clear cut framework within which to address the various effects of alternative schedules. These discussions are <u>not</u> academic exercises. They occur in contract negotiations, the Defense

Systems Acquisition Review Council process (DSARC), the Program Objective Memoranda process (POM), Cost Improvement Group (CAIG) meetings and contract disputes. Thus, increased understanding of production scheduling can lead to higher quality decisions about Air Force programs at all levels of the service.

C. <u>Program Management and Monitoring</u>

In addition to contributing to our general understanding of production scheduling, the revised model can be used for particular tasks in program management. For example, during contract negotiations the revised model may be used to evaluate a production rate variance formula simliar to the one in the A-10 contract (see Gaunt [9]). The contractor's proposed costs at alternative production rates can be compared to those forecasted by the model. This can be a significant basis for the negotiations about the clause. This is not to say that the model's forecasts are correct and the contractor's incorrect, but the model's forecast can serve as the basis for significant questions about why the contractor proposes a particular formula.

Likewise, the SPO can use the revised model as an aid in constructing the delivery schedule for the program. With a small amount of programming, the costs of many alternative delivery schedules can be quickly compared. These costs together with other considerations can be used to choose the best schedule.

Given the production schedule the SPO may use the model to layout a funding profile for the program. This plan could project costs by month by airframe for the entire program. As actual costs are reported and work packages are completed, this profile could serve as a check on program progress. Slips in the schedule could be spotted quickly and small problems identified before they become large problems. In this use, the model could play a role similar to the actual cost of work performed-budgeted cost of work performed comparisons that are now made.

The model can also provide a SPO with a quick response capability. Educating higher authority about the costs of changing production schedules will not eliminate all the proposals for change. As a result the SPO must be able to respond quickly and reliably to proposals for changes in delivery schedules and funding profiles. The revised model provides the means to forecast alternative cost profiles for different delivery schedules quickly and accurately. It can also be used to find a set of delivery schedules that fit a particular funding profile. Therefore it provides this quick reaction capability. This capability is demonstrated in Chapter V where five alternative delivery schedules were evaluated for the C-141 program.

D. Cost Estimation Techniques

This research has also produced airframe cost estimation techniques which are useful to the Air Force. In Chapter VI, two different techniques were derived. In Chapter VII these techniques were demonstrated on the F-5/T-38 program.

The first cost estimation technique provides a way to adjust the scaling factor on the C-141 model by using a cost estimating relation and design characteristics of the new airframe. This technique provides a way to estimate costs and funding profiles before any data is generated from the program. This technique is, therefore, ideal for use in independent cost analysis, AFSARC, CAIG and DSARC reports. The technique was applied to the T-38 data in Chapter VII. There we noted that the technique was primarily limited only by the accuracy of the cost estimating relation used in conjunction with the model.

A second cost estimation technique is the Bayesian updating technique. This method provides a way to update the estimated parameters of the model as early observations on the new program become available. While this technique would be used mainly by SPO personnel and those engaged in cost analysis for ongoing programs, it can also be applied in other circumstances. For example, a decision to reopen the C-5A production line or to restart the B-1 program could be nicely analyzed using this procedure. This procedure was also demonstrated on the T-38 data in Chapter VII.

E. Basis for Further Research

Still one more reason that this work is significant to the Air Force is the fact that it forms a strong basis for further research on production scheduling and program planning.

Certainly one area in which more work needs to be done is in the area of data consolidation. AFPRO's routinely collect data on airframe programs by month and by lot. This data is used for program management and by various defense audit agencies. Yet, for some reason, it does not seem to find its way to the ASD Cost Library.

The data that is collected there is almost all associated with cumulative labor hours by lot of airframes. In some instances, the labor hours are available by airframe, but the library does not to collect monthly data on airframe programs. As a result, it is very difficult to find data of sufficient quality to estimate the model accurately.

For this study, only the C-141 data was in a form which permitted adequate estimation of the model's parameters. The F-102 data required substantial and somewhat arbitrary transformation before it could be used. The T-38 data was of high quality, but it shed very little light on the timing of program costs because it was lot data. Nevertheless, the model based on the C-141 data provided a good fit to the F-102 and the T-38 data.

This success should provide the impetus to consolidate existing data from the AFPRO's at the cost library. With more appropriate test data, the ability of the model to perform should be enhanced. This follow on research would fit nicely with thesis research in AFIT's new program in cost analysis.

A second area for further research is the application of the model to other products; certainly engines and missles are good possibilities. It may very well be that tanks, ships, and ammunition would also be appropriate items for this type of modeling.

There are also three areas in which the model itself might be enhanced. First, more work needs to be done on multiple product production functions. This will permit a more complete linking of the airframes to each other in the model. This work is well started in Chapters III and IV, but it needs still more modeling work if it is to successfully describe the data that is available. A second are of enhancement is the application of the model to alternative contracts. The model is based on the assumption that the contractor is motivated to minimize discounted program costs. In fact, the contractor's motivation depends on the wording of the contract. In principle a model that is unique to the contract can be derived. This could form the basis for choosing among alternative contract types. It could also provide a substantive case for multi-year contracting. Work still needs to be done in this area.

Third, the model should be expanded to include hiring and firing costs. This will tend to slow down and smooth out the model's reaction to schedule changes. As it is now, the model tends to react a bit too quickly and too strongly to changes in delivery schedules.

F. Conclusions

This study is not the last word on airframe production planning or cost estimation. It represents but one more step in our understanding of the factors and forces that determine the costs of a production program. Nevertheless, at this stage, we can offer some hyotheses about these forces that are consistent with the data we have examined so far. While these hypotheses are not in any sense proved, they have been derived with the use of the C-141 data and confirmed by data on the F-102 and the T-38 airframes.

The first point to be made is that production scheduling does matter as a determinant of program cost. In Chapter V it is clear that even very small changes in the production schedule have an impact on the timing and the magnitude of program costs.

Second, it is important to realize that changes in the delivery schedule and in lot release dates cannot be easily summarized by a single variable like production rate or delivery rate. It seems clear that one of the reasons that past studies came to contradictory conclusions about the impact of production rate on production costs is that they asked the wrong question. We conclude that questions about the relation between production schedules and program costs require an examination of four production cost drivers; learning by doing, learning over time, production line speed and production line length.

Furthermore, we conclude that it is necessary to analyze the behavior of the contractor when developing cost models. Models which regard costs as mechanically related to other variables are destined to have problems in explaining real world data.

In addition, we find that the revised C-141 model is stable. It can be estimated reliably from early observations on an airframe program and used to predict later observations. The model also seems to be very stable among programs. When corrected for the scaling factor difference, a model estimated from one program does a reasonable job of predicting for a new program. Because of this, the Air Force needs to do a better job of maintaining and consolidating data on that will permit the model to be fully used. That is, Air Force data at the ASD

Cost Library should be augmented by montly data on aircraft programs. This data is regularly collected by AFPRO's and SPO's at the present time.

Finally, we observe that the Bayesian updating scheme provides an orderly method of incorporating new program information as it becomes available. This permits the model to be used at the beginning of a production program and then updated monthly as the program provides cost data.

Because of these encouraging results, we believe that the revised model of Chapter V is ready for use on Air Force programs.

Appendix A

C-141 Data Base

OBS	C	CT	CQ	TO	T1	D1	T2	D2	T3	D3	T4	D4	T5	D5	T6	D6	V
1	72088	1	0	0	5.8333	1	6.5000	2	6.8333	3	7.1667	4	7.5000	5	0	5	5.000
2	201565	2	0	0	5.8333	1	6.5000	2	6.8333	3	7.1667	4	7.5000	5	0	5	5.000
3	344319	3	0	0	5.8333	1	6.5000	2	6.8333	3	7.1667	4	7.5000	5	0	5	11.000
4	433270	4	0	0	5.8333	1	6.5000	2	6.8333	3	7.1667	4	7.5000	5	0	5	11.000
5	419499	5	0	0	5.8333	1	6.5000	2	6.8333	3	7.1667	4	7.5000	5	0	5	21.000
6	403188	6	0	0	5.8333	1	6.5000	2	6.8333	3	7.1667	4	7.5000	5	0	5	35.833
7	224715	7	0	0	5.8333	1	6.5000	2	6.8333	3	7.1667	4	7.5000	5	0	5	34.333
8	17483	8	0	0	5.8333	1	6.5000	2	6.8333	3	7.1667	4	7.5000	5	0	5	89.667
9	62116	3	5	2	7.5000	6	8.1667	8	9.1667	9	9.5000	10	9.8333	11	0	11	11.000
10	67736	4	5	2	7.5000	6	8.1667	8	9.1667	9	9.5000	10	9.8333	11	0	11	11.000
11	181232	5	5	2	7.5000	6	8.1667	8	9.1667	9	9.5000	10	9.8333	11	0	11	21.000
12	366108	6	5	2	7.5000	6	8.1667	8	9.1667	9	9.5000	10	9.8333	11	0	11	35.833
13	577447	7	5	2	7.5000	6	8.1667	8	9.1667	9	9.5000	10	9.8333	11	0	11	34.333
14	364834	8	5	2	7.5000	6	8.1667	8	9.1667	9	9.5000	10	9.8333	11	0	11	89.667
15	137385	9	5	2	7.5000	6	8.1667	8	9.1667	9	9.5000	10	9.8333	11	0	11	86.333
16	55549	10	5	2	7.5000	6	8.1667	8	9.1667	9	9.5000	10	9.8333	11	0	11	84.167
17	43339	5	11	4	9.8333	13	10.1667	14	10.5000	15	10.8333	18	11.1667	21	0	21	21.000
18	134836	6	11	4	9.8333	13	10.1667	14	10.5000	15	10.8333	18	11.1667	21	0	21	35.833
19	299240	7	11	4	9.8333	13	10.1667	14	10.5000	15	10.8333	18	11.1667	21	0	21	34.333
20	585699	8	11	4	9.8333	13	10.1667	14	10.5000	15	10.8333	18	11.1667	21	0	21	89.667
21	578984	9	11	4	9.8333	13	10.1667	14	10.5000	15	10.8333	18	11.1667	21	0	21	86.333
22	391490	10	11	4	9.8333	13	10.1667	14	10.5000	15	10.8333	18	11.1667	21	0	21	84.167
23	139154	11	11	4	9.8333	13	10.1667	14	10.5000	15	10.8333	18	11.1667	21	0	21	79.167
24	39182	12	11	4	9.8333	13	10.1667	14	10.5000	15	10.8333	18	11.1667	21	0	21	126.333
25	40438	6	21	5	11.5000	25	11.8333	32	12.1667	34	12.5000	36	0.0000	36	0	36	35.833
26	115425	7	21	5	11.5000	25	11.8333	32	12.1667	34	12.5000	36	0.0000	36	0	36	34.333
27	178189	8	21	5	11.5000	25	11.8333	32	12.1667	34	12.5000	36	0.0000	36	0	36	89.667
28	353538	9	21	5	11.5000	25	11.8333	32	12.1667	34	12.5000	36	0.0000	36	0	36	86.333
29	709079	10	21	5	11.5000	25	11.8333	32	12.1667	34	12.5000	36	0.0000	36	0	36	84.167
30	734591	11	21	5	11.5000	25	11.8333	32	12.1667	34	12.5000	36	0.0000	36	0	36	79.167
31	298276	12	21	5	11.5000	25	11.8333	32	12.1667	34	12.5000	36	0.0000	36	0	36	126.333
32	80049	13	21	5	11.5000	25	11.8333	32	12.1667	34	12.5000	36	0.0000	36	0	36	112.167
33	49689	8	36	7	12.5000	40	12.8333	47	13.1667	54	13.5000	64	13.8333	66	0	66	89.667
34	173357	9	36	7	12.5000	40	12.8333	47	13.1667	54	13.5000	64	13.8333	66	0	66	86.333
35	327439	10	36	7	12.5000	40	12.8333	47	13.1667	54	13.5000	64	13.8333	66	0	66	84.167
36	731824	11	36	7	12.5000	40	12.8333	47	13.1667	54	13.5000	64	13.8333	66	0	66	79.167
37	1395723	12	36	7	12.5000	40	12.8333	47	13.1667	54	13.5000	64	13.8333	66	0	66	126.333
38	943571	13	36	7	12.5000	40	12.8333	47	13.1667	54	13.5000	64	13.8333	66	0	66	112.167
39	175940	14	36	7	12.5000	40	12.8333	47	13.1667	54	13.5000	64	13.8333	66	0	66	161.333
40	58471	8	66	7	13.8333	71	14.1667	78	14.5000	85	14.8333	92	15.1667	94	0	94	89.667
41	15081	9	66	7	13.8333	71	14.1667	78	14.5000	85	14.8333	92	15.1667	94	0	94	86.333
42	33534	10	66	7	13.8333	71	14.1667	78	14.5000	85	14.8333	92	15.1667	94	0	94	84.167
43	179231	11	66	7	13.8333	71	14.1667	78	14.5000	85	14.8333	92	15.1667	94	0	94	79.167
44	354582	12	66	7	13.8333	71	14.1667	78	14.5000	85	14.8333	92	15.1667	94	0	94	126.333
45	1038170	13	66	7	13.8333	71	14.1667	78	14.5000	85	14.8333	92	15.1667	94	0	94	112.167
46	1062258	14	66	7	13.8333	71	14.1667	78	14.5000	85	14.8333	92	15.1667	94	0	94	161.333
47	212618	15	66	7	13.8333	71	14.1667	78	14.5000	85	14.8333	92	15.1667	94	0	94	135.500
48	125302	12	94	11	15.1667	100	15.5000	109	15.8333	118	16.1667	122	0.0000	122	0	122	126.333
49	288621	13	94	11	15.1667	100	15.5000	109	15.8333	118	16.1667	122	0.0000	122	0	122	112.167
50	855536	14	94	11	15.1667	100	15.5000	109	15.8333	118	16.1667	122	0.0000	122	0	122	161.333
51	989155	15	94	11	15.1667	100	15.5000	109	15.8333	118	16.1667	122	0.0000	122	0	122	135.500
52	219436	16	94	11	15.1667	100	15.5000	109	15.8333	118	16.1667	122	0.0000	122	0	122	179.333
53	52529	12	122	11	16.1667	127	16.5000	137	16.8333	145	17.1667	150	0.0000	150	0	150	126.333
54	47071	13	122	11	16.1667	127	16.5000	137	16.8333	145	17.1667	150	0.0000	150	0	150	112.167

OBS	C	CT	CQ	TO	T1	D1	T2	D2	T3	D3	T4	D4	T5	D5	T6	D6	V
55	259384	14	122	11	16.1667	127	16.5000	137	16.8333	145	17.1667	150	0.0000	150	0.0000	150	161.333
56	763601	15	122	11	16.1667	127	16.5000	137	16.8333	145	17.1667	150	0.0000	150	0.0000	150	135.500
57	910171	16	122	11	16.1667	127	16.5000	137	16.8333	145	17.1667	150	0.0000	150	0.0000	150	179.333
58	263585	17	122	11	16.1667	127	16.5000	137	16.8333	145	17.1667	150	0.0000	150	0.0000	150	152.167
59	61671	14	150	13	18.5000	155	18.8333	165	19.1667	173	19.5000	182	19.8333	184	0.0000	184	161.333
60	324027	15	150	13	18.5000	155	18.8333	165	19.1667	173	19.5000	182	19.8333	184	0.0000	184	135.500
61	607307	16	150	13	18.5000	155	18.8333	165	19.1667	173	19.5000	182	19.8333	184	0.0000	184	179.333
62	1197978	17	150	13	18.5000	155	18.8333	165	19.1667	173	19.5000	182	19.8333	184	0.0000	184	152.167
63	450802	18	150	13	18.5000	155	18.8333	165	19.1667	173	19.5000	182	19.8333	184	0.0000	184	134.833
64	36539	19	150	13	18.5000	155	18.8333	165	19.1667	173	19.5000	182	19.8333	184	0.0000	184	129.833
65	52429	20	150	13	18.5000	155	18.8333	165	19.1667	173	19.5000	182	19.8333	184	0.0000	184	106.333
66	38957	14	184	13	19.8333	191	20.1667	200	20.5000	209	20.8333	217	0.0000	217	0.0000	217	161.333
67	74744	15	184	13	19.8333	191	20.1667	200	20.5000	209	20.8333	217	0.0000	217	0.0000	217	135.500
68	255826	16	184	13	19.8333	191	20.1667	200	20.5000	209	20.8333	217	0.0000	217	0.0000	217	179.333
69	468110	17	184	13	19.8333	191	20.1667	200	20.5000	209	20.8333	217	0.0000	217	0.0000	217	152.167
70	1139031	18	184	13	19.8333	191	20.1667	200	20.5000	209	20.8333	217	0.0000	217	0.0000	217	134.833
71	474967	19	184	13	19.8333	191	20.1667	200	20.5000	209	20.8333	217	0.0000	217	0.0000	217	129.833
72	50261	20	184	13	19.8333	191	20.1667	200	20.5000	209	20.8333	217	0.0000	217	0.0000	217	106.333
73	53935	21	184	13	19.8333	191	20.1667	200	20.5000	209	20.8333	217	0.0000	217	0.0000	217	79.500
74	92356	16	217	15	20.8333	218	21.1667	228	21.5000	236	21.8333	245	22.1667	250	0.0000	250	179.333
75	265213	17	217	15	20.8333	218	21.1667	228	21.5000	236	21.8333	245	22.1667	250	0.0000	250	152.167
76	397664	18	217	15	20.8333	218	21.1667	228	21.5000	236	21.8333	245	22.1667	250	0.0000	250	134.833
77	936814	19	217	15	20.8333	218	21.1667	228	21.5000	236	21.8333	245	22.1667	250	0.0000	250	129.833
78	614624	20	217	15	20.8333	218	21.1667	228	21.5000	236	21.8333	245	22.1667	250	0.0000	250	106.333
79	83900	21	217	15	20.8333	218	21.1667	228	21.5000	236	21.8333	245	22.1667	250	0.0000	250	79.500
80	44822	22	217	15	20.8333	218	21.1667	228	21.5000	236	21.8333	245	22.1667	250	0.0000	250	52.167
81	52538	16	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	179.333
82	85632	17	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	152.167
83	253616	18	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	134.833
84	320644	19	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	129.833
85	772294	20	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	106.333
86	991115	21	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	79.500
87	355948	22	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	52.167
88	13316	23	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	25.500
89	5227	24	250	15	22.1667	254	22.5000	263	22.8333	272	23.1667	281	23.5000	283	23.8333	284	4.000

Appendix B

F102 Data Base

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
1	1	1	402475	1	5942	1
2	2	2	375849	1	5942	3
3	3	3	278963	2	5942	7
4	4	4	271223	2	5942	7
5	5	5	262498	2	5942	8
6	6	6	258078	2	5942	9
7	7	7	243726	2	5942	10
8	8	8	232766	2	5942	10
9	9	9	220833	2	5942	11
10	10	10	218827	2	5942	12
11	11	11	322447	3	5942	15
12	12	12	306736	3	5942	16
13	13	13	290470	3	5942	17
14	14	14	282951	3	5942	18
15	15	15	233125	4	5942	21
16	16	16	215379	4	5942	22
17	17	17	203122	4	5942	22
18	18	21	189770	4	5942	24
19	19	18	164120	4	5942	23
20	20	19	169080	4	5942	23
21	22	20	150387	4	5942	23
22	23	22	154606	4	5942	24
23	24	23	168896	4	5942	27
24	25	27	159485	4	5942	27
25	26	25	149109	4	5942	27
26	27	32	129792	5	5942	28
27	28	24	128958	5	5942	27
28	29	31	130389	5	5942	28
29	31	26	114872	4	23903	26
30	32	36	128470	5	5942	28
31	33	35	121932	5	5942	28
32	34	29	117969	5	5942	28
33	35	33	117364	5	5942	28
34	36	37	116895	5	5942	29
35	37	30	94174	5	23903	27
36	38	40	115466	5	5942	29
37	39	28	111137	5	5942	27
38	40	47	108890	5	5942	30
39	41	49	113487	5	5942	30
40	43	55	115846	5	5942	30
41	44	39	111029	5	5942	29
42	45	44	107397	5	5942	29
43	21	38	164751	4	5942	29
44	30	46	119597	5	5942	29
45	47	52	100520	6	23903	31
46	48	50	100504	6	23903	31
47	49	34	98316	6	23903	30
48	51	57	99317	6	23903	32
49	52	41	98162	6	23903	30
50	53	42	94614	6	23903	30
51	55	43	91628	6	23903	30
52	56	45	88098	6	23903	30
53	58	48	90164	6	23903	30
54	60	51	86840	6	23903	31

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
55	61	53	87197	6	23903	31
56	63	54	93812	6	23903	32
57	65	56	97725	6	23903	32
58	66	59	92401	7	23903	33
59	67	58	86481	7	23903	33
60	69	60	86830	7	23903	33
61	70	61	84477	7	23903	33
62	72	70	88589	7	23903	34
63	62	75	73200	9	23903	34
64	73	62	81738	7	23903	33
65	74	109	90762	7	23903	35
66	75	64	81865	7	23903	34
67	77	65	79839	7	23903	34
68	78	66	79358	7	23903	34
69	79	67	79651	7	23903	34
70	80	68	77606	7	23903	34
71	82	69	75097	7	23903	34
72	46	86	103137	6	23903	35
73	64	81	77297	9	23903	34
74	83	71	75510	7	23903	34
75	84	145	83796	7	23903	35
76	85	72	82662	7	23903	35
77	86	73	85787	7	23903	35
78	88	146	87891	7	23903	35
79	89	74	78264	7	23903	35
80	90	76	76967	7	23903	35
81	91	77	74963	7	23903	35
82	92	78	74628	7	23903	35
83	93	79	76608	7	23903	35
84	94	80	75689	7	23903	35
85	96	82	79580	8	29264	35
86	97	84	77425	8	29264	36
87	42	63	137990	6	23903	33
88	98	83	77409	8	29264	35
89	99	88	75854	8	29264	36
90	100	85	78187	8	29264	36
91	101	87	76044	8	29264	36
92	102	89	75017	8	29264	36
93	103	91	73337	8	29264	36
94	104	94	72091	8	29264	36
95	105	92	72916	8	29264	36
96	107	93	69718	8	29264	36
97	108	106	71085	8	29264	37
98	109	95	68818	8	29264	36
99	110	96	72137	8	29264	37
100	111	97	72101	8	29264	37
101	112	99	70917	8	29264	37
102	113	98	71333	8	29264	37
103	115	100	72966	8	29264	37
104	116	101	69869	8	29264	37
105	117	103	69895	8	29264	37
106	118	104	70448	8	29264	37
107	119	105	71490	8	29264	37
108	120	108	69560	8	29264	37

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
109	132	120	65290	9	29264	37
110	133	121	64415	9	29264	37
111	134	122	64269	9	29264	37
112	135	123	65396	9	29264	37
113	136	124	65398	9	29264	37
114	137	125	65747	9	29264	37
115	138	127	63262	9	29264	38
116	140	126	65332	9	29264	38
117	141	128	61274	9	29264	38
118	142	129	64319	9	29264	38
119	143	130	64793	9	29264	38
120	144	131	61270	9	29264	38
121	145	134	63046	9	29264	38
122	146	132	65928	9	29264	38
123	147	135	60844	9	29264	39
124	148	137	64753	9	29264	38
125	59	133	77679	9	23903	38
126	68	102	86933	9	23903	37
127	71	90	78669	9	23903	36
128	121	111	72530	9	29264	37
129	122	112	70642	9	29264	37
130	123	110	69385	9	29264	37
131	124	113	64465	9	29264	37
132	125	114	66052	9	29264	37
133	126	115	66023	9	29264	37
134	128	116	65284	9	29264	37
135	129	117	64232	9	29264	37
136	130	118	64695	9	29264	37
137	131	119	65503	9	29264	37
138	150	136	63054	9	29264	38
139	151	138	63048	9	29264	38
140	152	139	64929	9	29264	38
141	153	140	62522	9	29264	38
142	154	142	60538	9	29264	38
143	155	141	62861	9	29264	38
144	156	147	61370	9	29264	38
145	157	148	60747	9	29264	39
146	158	143	60665	9	29264	38
147	159	149	62824	9	29264	38
148	161	150	58087	9	29264	38
149	162	152	59473	9	29264	38
150	163	151	63951	9	29264	38
151	164	153	59320	9	29264	38
152	165	156	60055	9	29264	38
153	166	154	61220	9	29264	38
154	167	155	62458	9	29264	38
155	168	157	62412	9	29264	38
156	170	158	61843	9	29264	38
157	171	159	63077	9	29264	38
158	172	160	62071	9	29264	38
159	173	161	61858	10	29264	38
160	174	162	60979	10	29264	38
161	175	163	58349	10	29264	38
162	176	164	60204	10	29264	38

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
163	177	167	56691	10	29264	38
164	178	165	60527	10	29264	38
165	180	166	57210	10	29264	38
166	181	168	59645	10	29264	38
167	182	170	59433	10	29264	38
168	50	171	102354	6	23903	38
169	54	107	94057	8	23903	37
170	57	144	84495	8	23903	38
171	183	169	57653	10	29264	39
172	184	172	57566	10	29264	39
173	185	173	56691	10	29264	39
174	186	174	56621	10	29264	39
175	187	176	59791	10	29264	39
176	188	177	56079	10	29264	39
177	76	181	58506	10	23903	39
178	190	178	59994	10	29264	39
179	191	179	60757	10	29264	39
180	192	180	54645	10	29264	39
181	193	182	59312	10	29264	39
182	194	183	59483	10	29264	39
183	195	184	55581	10	29264	39
184	196	185	57455	10	29264	39
185	198	175	54241	10	29264	39
186	199	186	59042	10	29264	39
187	200	187	56001	10	29264	39
188	201	188	53381	10	29264	39
189	202	189	57139	10	29264	39
190	203	190	55403	10	29264	39
191	204	191	54271	10	29264	39
192	81	192	54031	10	23903	39
193	206	193	57332	10	29264	39
194	207	194	53560	10	29264	39
195	208	195	56839	10	29264	39
196	209	196	55439	10	29264	39
197	210	197	53354	10	29264	39
198	211	198	57169	10	29264	39
199	212	199	56625	10	29264	39
200	214	200	53698	10	29264	39
201	215	201	55117	10	29264	39
202	216	202	64410	11	31174	40
203	217	203	63457	11	31174	40
204	218	204	64855	11	31174	40
205	219	205	64709	11	31174	40
206	20	206	61493	11	31174	40
207	87	207	61633	11	23903	40
208	222	208	64154	11	31174	40
209	223	209	60020	11	31174	40
210	224	210	61057	11	31174	40
211	226	211	64077	11	31174	40
212	227	212	62750	11	31174	40
213	228	213	63673	11	31174	40
214	229	214	61986	11	31174	40
215	231	215	65932	11	31174	40
216	232	216	64059	11	31174	40

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
217	233	217	61146	11	31174	40
218	234	218	63329	11	31174	40
219	236	219	54223	12	31174	40
220	237	220	57406	12	31174	40
221	95	221	52714	11	23903	40
222	238	222	57999	12	31174	40
223	239	223	58051	12	31174	40
224	241	224	47604	12	31174	40
225	242	225	55653	12	31174	40
226	243	226	53132	12	31174	40
227	244	227	55019	12	31174	40
228	246	228	51793	12	31174	40
229	247	229	54301	12	31174	40
230	248	230	51429	12	31174	40
231	249	231	59606	12	31174	40
232	251	232	51181	12	31174	40
233	252	233	53072	12	31174	40
234	253	234	56653	12	31174	40
235	106	235	53057	11	23903	41
236	254	236	55143	12	31174	40
237	256	237	50899	12	31174	40
238	257	238	52395	12	31174	40
239	258	239	50183	12	31174	40
240	259	240	52217	12	31174	40
241	261	241	52656	12	31174	40
242	262	245	52818	12	31174	41
243	263	242	49485	12	31174	40
244	265	246	52511	12	31174	41
245	266	251	49671	12	31174	41
246	267	247	54124	12	31174	41
247	269	248	50204	12	31174	41
248	270	243	52963	12	31174	41
249	271	249	51224	12	31174	41
250	114	244	50005	11	23903	41
251	273	250	52793	12	31174	41
252	274	252	49813	12	31174	41
253	275	[illegible]	51312	12	31174	41
254	277	[illegible]	49304	12	31174	41
255	278	257	50879	12	31174	41
256	279	255	48798	12	31174	41
257	281	256	51624	12	31174	41
258	282	258	47738	12	31174	41
259	283	259	52435	12	31174	41
260	285	262	48038	12	31174	41
261	127	263	50935	12	23903	42
262	286	260	49978	12	31174	41
263	287	261	49255	12	31174	41
264	289	264	49755	12	31174	41
265	290	271	46508	12	31174	41
266	291	277	49555	12	31174	41
267	293	265	47469	12	31174	41
268	294	272	50337	12	31174	41
269	295	266	48865	12	31174	41
270	139	273	51644	12	23903	42

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
271	297	267	49463	12	31174	41
272	298	268	47582	12	31174	41
273	299	269	49975	12	31174	41
274	301	274	46285	12	31174	41
275	302	275	50195	12	31174	41
276	303	283	47662	12	31174	42
277	149	270	53718	13	29264	43
278	305	278	51616	12	31174	42
279	306	279	46355	12	31174	42
280	307	285	50820	12	31174	42
281	308	276	48181	12	31174	41
282	309	280	48972	12	31174	42
283	310	284	47284	12	31174	42
284	311	286	51997	12	31174	42
285	312	289	47163	12	31174	42
286	313	281	49759	12	31174	42
287	314	282	46257	12	31174	42
288	160	287	54706	13	29264	43
289	315	290	49545	12	31174	42
290	316	291	47045	12	31174	42
291	317	292	49222	12	31174	42
292	318	288	47705	12	31174	42
293	319	293	49419	12	31174	42
294	320	297	46921	12	31174	42
295	321	295	50453	12	31174	42
296	322	296	47644	12	31174	42
297	169	298	51951	13	29264	43
298	323	294	49567	12	31174	42
299	324	311	46926	13	31174	42
300	325	299	48883	13	31174	42
301	326	308	48317	13	31174	42
302	327	309	50921	13	31174	42
303	328	300	46097	13	31174	42
304	329	301	51757	13	31174	42
305	330	302	46997	13	31174	42
306	179	324	46463	14	29264	44
307	331	303	48965	13	31174	42
308	332	312	46431	13	31174	42
309	333	310	49284	13	31174	42
310	334	304	48369	13	31174	42
311	335	305	48940	13	31174	42
312	336	313	45578	13	31174	42
313	337	319	50382	13	31174	43
314	338	306	45772	13	31174	42
315	339	307	47439	13	31174	42
316	189	377	43656	14	29264	44
317	340	467	43759	13	31 74	46
318	341	314	49828	13	31174	42
319	342	315	46317	13	31174	42
320	343	316	48883	13	31174	42
321	344	320	46107	13	31174	43
322	345	317	47944	13	31174	43
323	346	321	46362	13	31174	43
324	347	318	49038	13	31174	43

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F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
325	348	325	48512	13	31174	43
326	349	322	48172	13	31174	43
327	350	323	45123	13	31174	43
328	197	365	44669	14	29264	44
329	351	326	46952	13	31174	43
330	352	327	45029	13	31174	43
331	353	331	47745	13	31174	43
332	354	328	43882	13	31174	43
333	355	329	46630	13	31174	43
334	356	330	46004	13	31174	43
335	357	332	48265	13	31174	43
336	358	333	45810	13	31174	43
337	359	336	47808	13	31174	43
338	360	348	45384	13	31174	43
339	205	384	41615	14	29264	45
340	361	334	47072	13	31174	43
341	362	337	46680	13	31174	43
342	363	335	45825	13	31174	43
343	364	338	46737	13	31174	43
344	365	349	44783	13	31174	43
345	366	339	46061	13	31174	43
346	367	340	46326	13	31174	43
347	368	347	44932	13	31174	43
348	369	341	46868	13	31174	43
349	370	342	45799	13	31174	43
350	213	385	40989	14	29264	45
351	371	343	45658	13	31174	43
352	372	353	44921	13	31174	43
353	373	350	45909	13	31174	43
354	374	344	45890	13	31174	43
355	375	345	45209	13	31174	43
356	376	346	46284	13	31174	43
357	377	351	44480	13	31174	43
358	378	354	46528	13	31174	43
359	379	355	45290	13	31174	43
360	380	352	46719	13	31174	43
361	220	366	43022	14	29264	44
362	381	356	45011	13	31174	43
363	382	357	46331	13	31174	43
364	383	358	45536	13	31174	43
365	384	397	45241	13	31174	44
366	385	359	44994	13	31174	43
367	386	360	46435	13	31174	43
368	387	361	44128	13	31174	43
369	388	367	47348	13	31174	44
370	389	362	43402	13	31174	43
371	390	368	47047	13	31174	44
372	391	369	44173	13	31174	44
373	225	410	40271	14	29264	46
374	392	363	46565	13	31174	44
375	393	370	44759	13	31174	44
376	394	364	45930	13	31174	44
377	395	378	46869	13	31174	44
378	396	373	46292	13	31174	44

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
379	397	380	44844	13	31174	44
380	398	371	44195	13	31174	44
381	399	372	45243	13	31174	44
382	400	386	45916	13	31174	44
383	401	375	44838	13	31174	44
384	402	381	44763	13	31174	44
385	230	421	41948	14	29264	45
386	403	374	44645	13	31174	44
387	404	376	43948	13	31174	44
388	405	389	43744	13	31174	44
389	406	382	44580	13	31174	44
390	407	391	46038	13	31174	44
391	408	387	43652	13	31174	44
392	409	379	42371	13	31174	44
393	410	383	45607	13	31174	44
394	411	388	44552	13	31174	44
395	412	392	47014	13	31174	44
396	413	393	44289	13	31174	44
397	414	394	45401	13	31174	44
398	235	428	39196	14	29264	46
399	415	390	43368	13	31174	44
400	416	403	46979	14	31174	44
401	417	398	44974	14	31174	44
402	418	404	46598	14	31174	44
403	419	395	43842	14	31174	44
404	420	399	46131	14	31174	44
405	421	396	43247	14	31174	44
406	422	400	44734	14	31174	44
407	423	401	43501	14	31174	44
408	424	402	46129	14	31174	44
409	425	405	42443	14	31174	44
410	240	454	34031	14	29264	46
411	426	417	44328	14	31174	45
412	427	406	44473	14	31174	44
413	428	407	43418	14	31174	44
414	429	408	43775	14	31174	44
415	430	409	42007	14	31174	44
416	431	411	42537	14	31174	44
417	432	422	43270	14	31174	45
418	433	418	42650	14	31174	45
419	434	412	43261	14	31174	45
420	435	413	42486	14	31174	45
421	436	414	42083	14	31174	45
422	437	419	42678	14	31174	45
423	245	440	34969	14	29264	46
424	438	425	41673	14	31174	45
425	439	415	41431	14	31174	45
426	440	420	41347	14	31174	45
427	441	416	43084	14	31174	45
428	442	429	42843	14	31174	45
429	443	462	44814	14	31174	45
430	444	441	41106	14	31174	45
431	445	423	41108	14	31174	45
432	446	426	41091	14	31174	45

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
433	447	424	42094	14	31174	45
434	448	430	40599	14	31174	45
435	250	470	33149	14	29264	46
436	449	437	43760	14	31174	45
437	450	427	42527	14	31174	45
438	451	438	41942	14	31174	45
439	452	431	41095	14	31174	45
440	453	432	42314	14	31174	45
441	454	466	40751	14	31174	45
442	455	434	40744	14	31174	45
443	456	439	39514	14	31174	45
444	457	433	41948	14	31174	45
445	458	442	39649	14	31174	45
446	459	435	41034	14	31174	45
447	460	436	38751	14	31174	45
448	255	475	32806	14	29264	46
449	461	443	41160	14	31174	45
450	462	444	39329	14	31174	45
451	463	445	40440	14	31174	45
452	464	446	38930	14	31174	45
453	465	447	39816	14	31174	45
454	466	455	40240	14	31174	45
455	467	448	39527	14	31174	45
456	468	449	38800	14	31174	45
457	469	450	40127	14	31174	45
458	470	456	38137	14	31174	45
459	471	451	40364	14	31174	45
460	472	559	35847	14	31174	48
461	265	499	34639	14	29264	47
462	473	452	36716	14	31174	45
463	474	457	38847	14	31174	45
464	475	458	37680	14	31174	45
465	476	459	39414	14	31174	45
466	477	453	36786	14	31174	45
467	478	476	41041	14	31174	46
468	479	471	38556	14	31174	46
469	480	460	38464	14	31174	45
470	481	477	38209	14	31174	46
471	482	463	39360	14	31174	45
472	483	461	39663	14	31174	45
473	484	464	39031	14	31174	45
474	485	465	36913	14	31174	45
475	486	472	39743	14	31174	46
476	487	468	38969	14	31174	46
477	264	480	32783	15	29264	46
478	488	469	37974	14	31174	46
479	489	481	37991	14	31174	46
480	490	482	37959	14	31174	46
481	491	473	38523	14	31174	46
482	492	478	37149	14	31174	46
483	493	474	38780	14	31174	46
484	494	479	38127	14	31174	46
485	495	490	38422	14	31174	46
486	496	483	37055	14	31174	46

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
487	497	491	38173	14	31174	46
488	498	492	36687	14	31174	46
489	499	484	38894	14	31174	46
490	500	485	37772	14	31174	46
491	258	537	33992	15	29264	47
492	501	486	39701	14	31174	46
493	502	487	36984	14	31174	46
494	503	488	37937	14	31174	46
495	504	493	39253	14	31174	46
496	505	494	39241	14	31174	46
497	506	495	39234	14	31174	46
498	507	504	34978	14	31174	47
499	508	489	39354	14	31174	46
500	509	496	36089	14	31174	46
501	510	502	38637	14	31174	46
502	511	497	36551	14	31174	46
503	512	498	37801	14	31174	46
504	513	508	37474	15	31174	47
505	514	500	39105	15	31174	46
506	515	505	36540	15	31174	47
507	272	546	34282	15	29264	47
508	516	501	39638	15	31174	46
509	517	509	37741	15	31174	47
510	518	506	40580	15	31174	47
511	519	507	38211	15	31174	47
512	520	510	37452	15	31174	47
513	521	514	39599	15	31174	47
514	522	511	34740	15	31174	47
515	523	527	39550	15	31174	47
516	524	515	39678	15	31174	47
517	525	516	37542	15	31174	47
518	526	503	39711	15	31174	46
519	527	512	40875	15	31174	47
520	276	557	33042	15	29264	47
521	528	517	39574	15	31174	47
522	529	528	34797	15	31174	47
523	530	518	39699	15	31174	47
524	531	519	40476	15	31174	47
525	532	520	40301	15	31174	47
526	533	530	39752	15	31174	47
527	534	523	37512	15	31174	47
528	535	513	42400	15	31174	47
529	536	560	37045	15	31174	48
530	537	529	41027	15	31174	47
531	538	521	39535	15	31174	47
532	539	524	39529	15	31174	47
533	280	555	31978	15	29264	47
534	540	522	38261	15	31174	47
535	541	525	40075	15	31174	47
536	542	538	36485	15	31174	44
537	543	531	39302	15	31174	47
538	544	526	39951	15	31174	47
539	545	532	36631	15	31174	47
540	546	533	37222	15	31174	47

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
541	547	534	38649	15	31174	47
542	548	566	35857	15	31174	48
543	549	535	39618	15	31174	47
544	284	540	31584	15	29264	47
545	550	541	38502	15	31174	47
546	551	536	39125	15	31174	47
547	552	551	37217	15	31174	48
548	553	542	40343	15	31174	47
549	554	539	39612	15	31174	47
550	555	544	39173	15	31174	47
551	556	552	37925	15	31174	48
552	557	547	39405	15	31174	47
553	558	543	39572	15	31174	47
554	559	572	38374	15	31174	48
555	288	564	33353	15	29264	48
556	560	549	39594	15	31174	48
557	561	550	36419	15	31174	48
558	562	553	39692	15	31174	48
559	563	545	38713	15	31174	47
560	564	554	37732	15	31174	48
561	565	548	40050	15	31174	47
562	566	556	39181	15	31174	48
563	567	581	38331	15	31174	49
564	568	558	40268	15	31174	48
565	292	574	33023	15	29264	48
566	569	567	36656	15	31174	48
567	570	561	41398	15	31174	48
568	571	562	39716	15	31174	48
569	572	569	37075	15	31174	48
570	573	563	40554	15	31174	48
571	574	570	39072	15	31174	48
572	575	596	35386	15	31174	49
573	576	565	38325	15	31174	48
574	577	568	39617	15	31174	48
575	578	571	35010	15	31174	48
576	296	593	32497	15	29264	48
577	579	575	38999	15	31174	48
578	580	585	36021	15	31174	49
579	581	576	39109	15	31174	48
580	582	573	38444	15	31174	48
581	583	582	34913	15	31174	49
582	584	583	38930	15	31174	49
583	300	598	32088	15	29264	48
584	585	603	35935	15	31174	49
585	586	584	38656	15	31174	49
586	587	586	38330	15	31174	49
587	588	587	36771	15	31174	49
588	589	608	38779	15	31174	49
589	590	594	37715	15	31174	49
590	591	604	34194	15	31174	49
591	304	618	32235	15	29264	48
592	592	599	37819	15	31174	49
593	593	589	35745	15	31174	49
594	594	579	38478	15	31174	48

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
595	595	577	35462	15	31174	48
596	596	578	39201	15	31174	48
597	597	631	46899	16	31174	49
598	598	580	38757	15	31174	48
599	599	588	35842	15	31174	49
600	600	590	39562	15	31174	49
601	601	591	34902	15	31174	49
602	602	592	39059	15	31174	49
603	603	595	35372	15	31174	49
604	604	605	35354	15	31174	49
605	605	597	39369	15	31174	49
606	606	674	47958	16	31174	50
607	607	614	35772	15	31174	49
608	608	606	39562	15	31174	49
609	609	600	33732	15	31174	49
610	610	609	34407	15	31174	49
611	611	601	38240	15	31174	49
612	612	623	47112	16	31174	49
613	613	602	35079	15	31174	49
614	614	607	38041	16	31174	49
615	615	610	36404	16	31174	49
616	616	611	34422	16	31174	49
617	617	615	36994	16	31174	49
618	618	626	35568	16	31174	50
619	619	659	38261	16	31174	49
620	620	612	37388	16	31174	49
621	621	613	34891	16	31174	49
622	622	650	37237	16	31174	50
623	623	616	37872	16	31174	49
624	624	668	34862	16	31174	50
625	625	651	36136	16	31174	49
626	626	617	37410	16	31174	49
627	627	627	35840	16	31174	50
628	628	637	37068	16	31174	50
629	629	619	38618	16	31174	50
630	630	629	34671	16	31174	50
631	631	634	36356	16	31174	50
632	632	628	37722	16	31174	50
633	633	676	36013	16	31174	50
634	634	624	35462	16	31174	50
635	635	640	38700	16	31174	50
636	636	621	36422	16	31174	50
637	637	625	37465	16	31174	50
638	638	622	37141	16	31174	50
639	639	666	34955	16	31174	50
640	640	638	35274	16	31174	50
641	641	620	38790	16	31174	50
642	642	630	33908	16	31174	50
643	643	657	34959	16	31174	50
644	644	632	39424	16	31174	50
645	645	641	35521	16	31174	50
646	646	639	34366	16	31174	49
647	647	635	37056	16	31174	50
648	648	642	34659	16	31174	50

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
649	649	643	34626	16	31174	50
650	650	633	38519	16	31174	50
651	651	644	34933	16	31174	50
652	652	694	36773	16	31174	50
653	653	636	36636	16	31174	50
654	654	646	34392	16	31174	50
655	655	652	35328	16	31174	50
656	656	647	37233	16	31174	50
657	657	653	34241	16	31174	50
658	658	658	35014	16	31174	50
659	659	696	35611	16	31174	50
660	660	654	36221	16	31174	50
661	661	645	34440	16	31174	50
662	662	648	34129	16	31174	50
663	663	655	35552	16	31174	50
664	664	649	35529	16	31174	50
665	665	656	34409	16	31174	50
666	666	660	35921	16	31174	49
667	667	661	36223	16	31174	50
668	668	663	35645	16	31174	50
669	669	664	34681	16	31174	50
670	670	669	33087	16	31174	50
671	671	662	32965	16	31174	50
672	672	681	35670	16	31174	51
673	673	704	36601	16	31174	50
674	674	665	35246	16	31174	50
675	675	675	34594	16	31174	51
676	676	670	34931	16	31174	51
677	677	671	36060	16	31174	51
678	678	667	34677	16	31174	50
679	679	672	35208	16	31174	50
680	680	673	37208	16	31174	51
681	681	677	34359	16	31174	51
682	682	678	37355	16	31174	51
683	683	679	34438	16	31174	51
684	684	682	37074	16	31174	51
685	685	710	36071	16	31174	51
686	686	713	37780	16	31174	51
687	687	683	35409	16	31174	51
688	688	680	34491	16	31174	51
689	689	686	34787	16	31174	51
690	690	684	35095	16	31174	51
691	691	685	36314	16	31174	51
692	692	687	36223	16	31174	51
693	693	718	34963	16	31174	51
694	694	691	34156	16	31174	51
695	695	688	34344	16	31174	51
696	696	689	35611	16	31174	51
697	697	692	36574	16	31174	51
698	698	693	36607	16	31174	51
699	699	706	33850	16	31174	51
700	700	690	34392	16	31174	51
701	701	697	34865	16	31174	51
702	702	705	34577	16	31174	51

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
703	703	700	34977	16	31174	51
704	704	695	34248	16	31174	51
705	705	755	35548	16	31174	51
706	706	698	34095	16	31174	51
707	707	701	35518	16	31174	51
708	708	699	35575	16	31174	51
709	709	757	33410	16	31174	52
710	710	720	33536	16	31174	52
711	711	711	34114	16	31174	51
712	712	776	32634	16	31174	51
713	713	707	37553	16	31174	51
714	714	714	34267	16	31174	51
715	715	708	34312	16	31174	51
716	716	709	33612	16	31174	51
717	717	715	35161	16	31174	52
718	718	806	30854	16	31174	54
719	719	702	33967	16	31174	51
720	720	737	36243	16	31174	52
721	721	716	33337	16	31174	52
722	722	717	34825	16	31174	52
723	723	703	33320	16	31174	51
724	724	758	34063	17	31174	51
725	725	721	36791	16	31174	52
726	726	722	36188	16	31174	52
727	727	723	34847	16	31174	52
728	728	712	33222	16	31174	51
729	729	719	33823	17	31174	52
730	730	777	33796	17	31174	53
731	731	728	29287	17	31174	52
732	732	724	36611	17	31174	52
733	733	727	34881	17	31174	52
734	734	725	35445	17	31174	52
735	735	729	33745	17	31174	52
736	736	778	34022	17	31174	53
737	737	742	37557	17	31174	52
738	738	726	34634	17	31174	52
739	739	732	34078	17	31174	52
740	740	734	36481	17	31174	52
741	741	733	35237	17	31174	52
742	742	789	33297	17	31174	54
743	743	730	36853	17	31174	52
744	744	735	37685	17	31174	52
745	745	731	35273	17	31174	52
746	746	736	35942	17	31174	52
747	747	743	35034	17	31174	52
748	748	780	31739	17	31174	53
749	749	738	38371	17	31174	52
750	750	740	35908	17	31174	52
751	751	739	36085	17	31174	52
752	752	744	37608	17	31174	52
753	753	741	34999	17	31174	52
754	754	784	32386	18	31174	53
755	755	745	35262	17	31174	52
756	756	746	38751	17	31174	52

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
757	757	747	38249	17	31174	52
758	758	748	34400	17	31174	52
759	759	749	35699	17	31174	52
760	760	781	30522	18	31174	53
761	761	750	36158	17	31174	52
762	762	759	37197	17	31174	52
763	763	751	34457	17	31174	52
764	764	752	34742	17	31174	52
765	765	753	35335	17	31174	52
766	766	785	31246	18	31174	54
767	767	756	40435	17	31174	52
768	768	760	35629	17	31174	52
769	769	754	35135	17	31174	52
770	770	764	35231	17	31174	52
771	771	765	34874	17	31174	52
772	772	802	31404	18	31174	54
773	773	761	41244	17	31174	52
774	774	766	36414	17	31174	53
775	775	762	34569	17	31174	52
776	776	763	35179	17	31174	52
777	777	767	35442	17	31174	53
778	778	798	30783	18	31174	54
779	779	768	36051	17	31174	53
780	780	772	33038	17	31174	53
781	781	769	35534	17	31174	53
782	782	770	35803	17	31174	53
783	783	771	35196	17	31174	53
784	784	793	30547	18	31174	54
785	785	773	34112	17	31174	53
786	786	774	34077	17	31174	53
787	787	779	35163	17	31174	53
788	788	786	36591	17	31174	53
789	789	787	34417	17	31174	53
790	790	796	29910	18	31174	54
791	791	775	34673	17	31174	53
792	792	782	37329	17	31174	53
793	793	790	33861	17	31174	53
794	794	783	33740	17	31174	53
795	795	788	34446	17	31174	53
796	796	808	29780	18	31174	55
797	797	791	34881	17	31174	53
798	798	792	37439	17	31174	53
799	799	794	34056	17	31174	53
800	800	795	35196	17	31174	53
801	801	803	33631	17	31174	53
802	802	809	30199	18	31174	55
803	803	797	33470	17	31174	53
804	804	799	33401	17	31174	53
805	805	800	36033	17	31174	53
806	806	801	34735	17	31174	53
807	807	807	35964	17	31174	53
808	808	812	30414	18	31174	55
809	809	804	35076	17	31174	53
810	810	805	34456	17	31174	53

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
811	811	810	33894	17	31174	53
812	812	811	32604	17	31174	53
813	813	818	33699	17	31174	53
814	814	833	30809	18	31174	55
815	815	816	34827	17	31174	53
816	816	813	33166	17	31174	53
817	817	814	33378	17	31174	53
818	818	815	35735	17	31174	53
819	819	817	33344	17	31174	53
820	820	830	30231	18	31174	55
821	821	819	33827	17	31174	53
822	822	820	33591	17	31174	53
823	823	821	34865	17	31174	53
824	824	822	33164	17	31174	53
825	825	823	31263	17	31174	54
826	826	831	29503	18	31174	55
827	827	824	32246	17	31174	54
828	828	825	33635	17	31174	54
829	829	828	34551	17	31174	54
830	830	826	32501	17	31174	54
831	831	832	31916	17	31174	54
832	832	866	30404	18	31174	55
833	833	829	32252	17	31174	54
834	834	834	31902	17	31174	54
835	835	841	33996	17	31174	54
836	836	860	28680	18	31174	54
837	837	827	37834	18	33695	54
838	838	835	33819	18	33695	54
839	839	836	34434	18	33695	54
840	840	837	34145	18	33695	54
841	841	868	26963	18	31174	55
842	842	843	34026	18	33695	54
843	843	842	35753	18	33695	54
844	844	838	35514	18	33695	54
845	845	839	35737	18	33695	54
846	846	872	27959	18	31174	55
847	847	847	35407	18	33695	54
848	848	853	33939	18	33695	54
849	849	840	35909	18	33695	54
850	850	844	34741	18	33695	54
851	851	885	27842	18	31174	56
852	852	845	32823	18	33695	54
853	853	846	35274	18	33695	54
854	854	851	33355	18	33695	54
855	855	852	33079	18	33695	54
856	856	886	28117	18	31174	56
857	857	848	36513	18	33695	54
858	858	849	35299	18	33695	54
859	859	850	33582	18	33695	54
860	860	858	34777	18	33695	54
861	861	887	27597	18	31174	56
862	862	854	34546	18	33695	54
863	863	855	32407	18	33695	54
864	864	861	33660	18	33695	54

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
865	865	856	34397	18	33695	54
866	866	901	28521	18	31174	56
867	867	859	34846	18	33695	54
868	868	857	35139	18	33695	54
869	869	862	33760	18	33695	54
870	870	864	33211	18	33695	54
871	871	902	27363	18	31174	56
872	872	865	32836	18	33695	54
873	873	869	32974	18	33695	54
874	874	870	34568	18	33695	55
875	875	903	27445	18	31174	56
876	876	863	32998	18	33695	54
877	877	867	37656	18	33695	54
878	878	871	35504	18	33695	55
879	879	873	33427	18	33695	55
880	880	911	27206	18	31174	56
881	881	882	34010	18	33695	55
882	882	874	33734	18	33695	55
883	883	875	34359	18	33695	55
884	884	915	27962	18	31174	56
885	885	876	32819	18	33695	55
886	886	877	33989	18	33695	55
887	887	878	35137	18	33695	55
888	888	937	27810	19	31174	57
889	889	879	32829	18	33695	55
890	890	880	34972	18	33695	55
891	891	881	34676	18	33695	55
892	892	921	26843	19	31174	56
893	893	883	34953	18	33695	55
894	894	884	33597	18	33695	55
895	895	888	33743	18	33695	55
896	896	922	27246	19	31174	57
897	897	889	33774	18	33695	55
898	898	894	33776	18	33695	55
899	899	891	35005	18	33695	55
900	900	925	27594	19	31174	57
901	901	890	34229	18	33695	55
902	902	895	34168	18	33695	55
903	903	896	33134	18	33695	55
904	904	940	26532	19	31174	57
905	905	892	35520	18	33695	55
906	906	897	37081	18	33695	55
907	907	909	33940	18	33695	56
908	908	941	26868	19	31174	57
909	909	904	34203	18	33695	55
910	910	898	33403	18	33695	55
911	911	899	35234	18	33695	55
912	912	951	26066	19	31174	57
913	913	893	32600	18	33695	55
914	914	905	33995	18	33695	55
915	915	906	35752	18	33695	55
916	916	952	26963	19	31174	57
917	917	900	34502	18	33695	55
918	918	912	36888	18	33695	56

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
919	919	907	34899	18	33695	55
920	920	959	26924	19	31174	58
921	921	917	33015	18	33695	56
922	922	908	35083	18	33695	56
923	923	910	34338	18	33695	56
924	924	963	27539	19	31174	58
925	925	913	35701	18	33695	56
926	926	916	34399	18	33695	56
927	927	914	34862	18	33695	56
928	928	956	26936	19	31174	57
929	929	923	34199	18	33695	56
930	930	918	33623	18	33695	56
931	931	919	33184	18	33695	56
932	932	920	35523	18	33695	56
933	933	968	26664	19	31174	58
934	934	926	32594	18	33695	56
935	935	924	35638	18	33695	56
936	936	927	33121	18	33695	56
937	937	928	34661	18	33695	56
938	938	929	35823	18	33695	56
939	939	964	27609	19	31174	58
940	940	930	33106	18	33695	56
941	941	934	34416	18	33695	56
942	942	938	33614	18	33695	56
943	943	931	34184	18	33695	56
944	944	932	35173	18	33695	56
945	945	933	33130	18	33695	56
946	946	969	27583	19	31174	58
947	947	935	33161	19	33695	56
948	948	942	35352	19	33695	57
949	949	936	33524	19	33695	56
950	950	939	34562	19	33695	56
951	951	943	33249	19	33695	57
952	952	944	34526	19	33695	57
953	953	945	33298	19	33695	57
954	954	946	35986	19	33695	57
955	955	953	33811	19	33695	57
956	956	947	35234	19	33695	57
957	957	948	34162	19	33695	57
958	958	954	35318	19	33695	57
959	959	949	34311	19	33695	57
960	960	957	35146	19	33695	57
961	961	950	33557	19	33695	57
962	962	955	34874	19	33695	57
963	963	958	33431	19	33695	57
964	964	960	34452	19	33695	57
965	965	961	33852	19	33695	57
966	966	962	34179	19	33695	57
967	967	965	33625	19	33695	57
968	968	966	34513	19	33695	57
969	969	967	33883	19	33695	57
970	970	970	35719	19	33695	58
971	971	971	33158	19	33695	58
972	972	972	34909	19	33695	58

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
919	919	907	34899	18	33695	55
920	920	959	26924	19	31174	58
921	921	917	33015	18	33695	56
922	922	908	35083	18	33695	56
923	923	910	34338	18	33695	56
924	924	963	27539	19	31174	58
925	925	913	35701	18	33695	56
926	926	916	34399	18	33695	56
927	927	914	34862	18	33695	56
928	928	956	26936	19	31174	57
929	929	923	34199	18	33695	56
930	930	918	33623	18	33695	56
931	931	919	33184	18	33695	56
932	932	920	35523	18	33695	56
933	933	968	26664	19	31174	58
934	934	926	32594	18	33695	56
935	935	924	35638	18	33695	56
936	936	927	33121	18	33695	56
937	937	928	34661	18	33695	56
938	938	929	35823	18	33695	56
939	939	964	27609	19	31174	58
940	940	930	33106	18	33695	56
941	941	934	34416	18	33695	56
942	942	938	33614	18	33695	56
943	943	931	34184	18	33695	56
944	944	932	35173	18	33695	56
945	945	933	33130	18	33695	56
946	946	969	27583	19	31174	58
947	947	935	33161	19	33695	56
948	948	942	35352	19	33695	57
949	949	936	33524	19	33695	56
950	950	939	34562	19	33695	56
951	951	943	33249	19	33695	57
952	952	944	34526	19	33695	57
953	953	945	33298	19	33695	57
954	954	946	35986	19	33695	57
955	955	953	33811	19	33695	57
956	956	947	35234	19	33695	57
957	957	948	34162	19	33695	57
958	958	954	35318	19	33695	57
959	959	949	34311	19	33695	57
960	960	957	35146	19	33695	57
961	961	950	33557	19	33695	57
962	962	955	34874	19	33695	57
963	963	958	33431	19	33695	57
964	964	960	34452	19	33695	57
965	965	961	33852	19	33695	57
966	966	962	34179	19	33695	57
967	967	965	33625	19	33695	57
968	968	966	34513	19	33695	57
969	969	967	33883	19	33695	57
970	970	970	35719	19	33695	58
971	971	971	33158	19	33695	58
972	972	972	34909	19	33695	58

F102 PROGRAM COST HISTORY

OBS	PLN	DELYSEQ	THOURS	LOT	CONTRACT	DM
973	973	973	35190	[illegible]	33695	58
974	974	974	35996	[illegible]	33695	58
975	975	980	34726	19	33695	58
976	976	975	36536	19	33695	58
977	977	976	35178	19	33695	58
978	978	977	35928	19	33695	58
979	979	978	34608	19	33695	58
980	980	981	35039	19	33695	58
981	981	979	34871	19	33695	58
982	982	982	35962	19	33695	58
983	983	983	34903	19	33695	58
984	984	984	35568	19	33695	59
985	985	985	35410	19	33695	59
986	986	988	35086	19	33695	59
987	987	986	34918	19	33695	59
988	988	992	37208	19	33695	59
989	989	987	35710	19	33695	59
990	990	989	36059	19	33695	59
991	991	995	35739	19	33695	59
992	992	990	37430	19	33695	60
993	993	996	37169	19	33695	59
994	994	993	35521	19	33695	60
995	995	997	38659	19	33695	59
996	996	991	37916	19	33695	59
997	997	994	38443	19	33695	60
998	998	998	40542	19	33695	60
999	999	999	43231	19	33695	60
1000	1000	1000	46441	19	33695	60

Appendix C

T-38 Data Base

OBS	L	C	CQ	T0	T1	D1	T2	D2	T3	D3	T4	D4
1	1	2.02826	0	0.50	18.5	1	19.5	2	0.0	0	0.0	0
2	2	1.49273	2	2.50	20.5	3	21.5	4	0.0	0	0.0	0
3	3	2.47343	4	8.50	22.5	5	23.5	6	24.5	7	25.5	8
4	4	2.13778	8	9.50	26.5	9	27.5	10	28.5	11	29.5	12
5	5	2.86235	12	9.50	30.5	13	31.5	14	32.5	16	33.5	17
6	6	5.00356	17	23.53	33.5	20	34.5	23	35.5	27	36.5	31
7	7	4.30792	32	25.90	37.5	35	38.5	41	39.5	48	0.0	0
8	8	4.57162	48	28.80	40.5	57	41.5	67	0.0	0	0.0	0
9	9	7.89551	67	31.30	42.5	79	43.5	91	44.5	103	0.0	0
10	10	6.68408	103	35.03	45.5	115	46.5	126	47.5	138	48.5	139
11	11	6.40262	139	38.03	48.5	151	49.5	163	50.5	175	0.0	0
12	12	6.06082	175	41.03	51.5	187	52.5	199	53.5	211	0.0	0
13	13	5.76845	211	43.33	54.5	223	55.5	235	56.5	247	0.0	0
14	14	5.53932	247	46.30	57.5	259	58.5	271	59.5	283	0.0	0
15	15	5.13834	283	49.87	60.5	295	61.5	307	62.5	319	0.0	0
16	16	4.94495	319	52.20	63.5	331	64.5	343	65.5	355	0.0	0
17	17	4.77346	355	54.40	66.5	367	67.5	379	68.5	391	0.0	0
18	18	4.57332	391	57.27	69.5	403	70.5	415	71.5	427	0.0	0
19	19	4.43431	427	60.87	72.5	439	73.5	451	74.5	463	0.0	0
20	20	3.59577	463	64.53	75.5	475	76.5	487	77.5	492	0.0	0
21	21	5.38563	492	67.13	77.5	499	78.5	511	79.5	523	80.5	535
22	22	5.15681	539	70.77	81.5	547	82.5	559	83.5	571	84.5	583
23	23	5.03100	585	74.57	85.5	595	86.5	607	87.5	619	88.5	631
24	24	5.02396	632	77.63	89.5	644	90.5	657	91.5	670	92.5	680
25	25	4.97425	680	81.63	92.5	684	93.5	697	94.5	710	95.5	723

OBS	T5	D5	T6	D6	T7	D7	T8	D8	T9	D9	V
1	0.0	0	0.0	0	0.0	0	0.0	0	0.0	2	10.444
2	0.0	0	0.0	0	0.0	0	0.0	0	0.0	4	11.833
3	0.0	0	0.0	0	0.0	0	0.0	0	0.0	8	18.438
4	0.0	0	0.0	0	0.0	0	0.0	0	0.0	12	22.365
5	0.0	0	0.0	0	0.0	0	0.0	0	0.0	17	30.870
6	37.5	32	0.0	0	0.0	0	0.0	0	0.0	32	57.107
7	0.0	0	0.0	0	0.0	0	0.0	0	0.0	48	80.952
8	0.0	0	0.0	0	0.0	0	0.0	0	0.0	67	98.376
9	0.0	0	0.0	0	0.0	0	0.0	0	0.0	103	120.820
10	0.0	0	0.0	0	0.0	0	0.0	0	0.0	139	137.370
11	0.0	0	0.0	0	0.0	0	0.0	0	0.0	175	146.207
12	0.0	0	0.0	0	0.0	0	0.0	0	0.0	211	152.049
13	0.0	0	0.0	0	0.0	0	0.0	0	0.0	247	160.805
14	0.0	0	0.0	0	0.0	0	0.0	0	0.0	283	167.951
15	0.0	0	0.0	0	0.0	0	0.0	0	0.0	319	168.646
16	0.0	0	0.0	0	0.0	0	0.0	0	0.0	355	179.675
17	0.0	0	0.0	0	0.0	0	0.0	0	0.0	391	192.214
18	0.0	0	0.0	0	0.0	0	0.0	0	0.0	427	202.646
19	0.0	0	0.0	0	0.0	0	0.0	0	0.0	463	220.111
20	0.0	0	0.0	0	0.0	0	0.0	0	0.0	492	241.520
21	81.5	539	0.0	0	0.0	0	0.0	0	0.0	539	279.655
22	85.5	586	0.0	0	0.0	0	0.0	0	0.0	586	286.672
23	89.5	632	0.0	0	0.0	0	0.0	0	0.0	632	284.135
24	0.0	0	0.0	0	0.0	0	0.0	0	0.0	680	269.863
25	96.5	728	0.0	0	0.0	0	0.0	0	0.0	728	234.679

BS	L	C	CQ	T0	T1	D1	T2	D2	T3	D3	T4	D4
6	26	6.47099	728	85.50	96.5	737	97.5	750	98.5	763	99.5	776
7	27	3.21258	792	88.50	101.5	803	102.5	815	103.5	821	0.0	0
8	28	5.27816	821	92.50	103.5	824	104.5	833	105.5	841	106.5	848
9	29	4.68911	872	96.50	110.5	876	111.5	883	112.5	890	113.5	897
0	31	4.09679	918	104.03	117.5	925	118.5	932	119.5	939	120.5	946
1	32	4.87972	953	107.03	122.5	961	123.5	970	124.5	979	125.5	988
2	33	7.78611	997	112.03	127.5	1008	128.5	1019	129.5	1030	130.5	1041
3	35	6.12747	1066	120.03	133.5	1074	134.5	1085	135.5	1096	136.5	1105
4	37	2.00109	1120	128.50	142.5	1122	143.5	1124	144.5	1126	145.5	1128
5	39	2.18518	1136	135.50	150.5	1138	151.5	1140	152.5	1143	153.5	1147
6	40	2.42932	1156	139.50	156.5	1159	157.5	1163	158.5	1167	159.5	1171
7	41	0.92816	1179	145.00	162.5	1183	163.5	1187	0.0	0	0.0	0

BS	T5	D5	T6	D6	T7	D7	T8	D8	T9	D9	V
6	100.5	790	101.5	792	0.0	0	0.0	0	0.0	792	201.467
7	0.0	0	0.0	0	0.0	0	0.0	0	0.0	221	179.143
8	107.5	855	108.5	862	109.5	869	110.5	872	0.0	872	163.471
9	114.5	904	115.5	911	116.0	918	0.0	0	0.0	918	161.839
0	121.5	953	0.0	0	0.0	0	0.0	0	0.0	953	187.614
1	126.5	997	0.0	0	0.0	0	0.0	0	0.0	997	195.885
2	131.5	1052	132.5	1063	133.5	1066	0.0	0	0.0	1066	188.862
3	137.5	1111	138.5	1114	139.5	1116	140.5	1118	141.5	1120	149.389
4	146.5	1130	147.5	1132	148.5	1134	149.5	1136	0.0	1136	106.850
5	154.5	1151	155.5	1155	156.5	1156	0.0	0	0.0	1156	83.853
6	160.5	1175	161.5	1179	0.0	0	0.0	0	0.0	1179	71.239
7	0.0	0	0.0	0	0.0	0	0.0	0	0.0	1187	58.763

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